

Vectorial Lorentz Transformations

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Abstract: We have noticed in relativistic literature that the derivation of Lorentz Transformations (LT) usually is presented by confining the moving system O' to move along the X-axis, namely, as a particular case of a more general movement. When this movement is generalized different transformations are obtained (which is a contradiction by itself) and a hidden vectorial characteristic of time is revealed. LT have been generalized in order to solve some physical and mathematical inconsistencies, such as the dissimilar manners (transversal, longitudinal) the particle's shape is influenced by its velocity and LT's inconsistency with Maxwell equations when in its derivation the pulse of light is sent perpendicular to the displacement of the moving system O'. Unlike the canonical derivation of LT, in the undertaken development of the generalized LT, assumptions were not used. Practical applications of generalized Vectorial Lorentz Transformations (VLT) were undertaken and as outcome a new definition of Local Lorentz Transformations (LLT) of magnitudes appeared. As another consequence, a characteristic and unique scaling Lorentz factor was obtained for each magnitude. Given this, a dimensional analysis based upon these Lorentz factors came up. In addition, dynamical transformations were obtained and a new definition of mass was found.

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1. Introduction

The unexpected results obtained by Michelson and Morley in 1881, according to which Earth did not move in any direction, under the interpretation induced at that time by Ether hypothesis, motivated Dutch physicist Hendrick Antoon Lorentz and British physicist George Francis FitzGerald, almost simultaneously during 1889-1890, to look for

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a mathematical and physical way to interpret this result. They undertook this task by preserving Maxwell equations to be the same in any inertial system, and by taking into account the constancy of the speed of light by observers with different inertial movements. This led to discard the undoubted, untouchable and accepted, until that time, Galilean transformations, and to replace them by Lorentz Transformations (LT) in 1904, when they were displayed formally [1]. However, it was known at that time that Galilean transformations although privileged as of “physical common sense”, neither preserved the constancy of light speed nor were consistent with Maxwell Equations. After Einstein’s solid arguments destroyed Ether hypothesis in 1905 [2], LT became central for the Special Theory of Relativity (STR) [3]. Nevertheless, LT, as we previously indicate, have their limitations.

This work was focused in showing these limitations, in order to derive a way to eliminate them and start developing a more general approach of LT. The master key for achieving this task was the presentation of LT under a generalized configuration of an inertial stationary system and another moving system at a constant velocity, along an inclined line not coincident with any axis (**Fig. A5, Annex 1**). In this general case both systems were considered as orthogonal systems, with all their corresponding axes being parallel. It can be shown that the usual one-dimensional (extended by isotropy to other coordinates) configuration modernly used to present the LT does not allow recognizing such limitations of LT. It should be noted that Lorentz did his work based on a system moving along the X-axis by assuming the coordinates y' and z' as not depending on the Lorentz’s scaling factors, namely, $y' = y$ and $z' = z$ [1]. Almost Independently, Einstein presented this derivation more directly, but assuming the same [2]. Since then, this has become the canonical way of presenting LT.

The generalized configuration used in this work took to us to obtain transformations, different from those given by the well-known LT, without doing any type of assumptions. It also revealed to us **a vectorial presentation of time** with components **depending on spatial coordinates** within such transformations. This surprising detection was, therefore, a direct consequence of the different configuration and procedure used.

It is worth mentioning that time, having vectorial properties, has recently been proposed, as a new theory or hypothesis for solving spin problems, by several authors including E. A. B. Cole [5-7], J. Strnad [8-9], V. Barashenkov [11-13], Xiadong Cheng [14], A. P. Yefremov [15], H. Kitada [16-17], J. E. Carroll [18], among others. A common point that these authors considered is a multi-dimensional vector time, with components **not depending** on spatial coordinates, under the canonical Lorentzian scheme of presentation previously referred. In order to be consistent with currently accepted relativity concepts, channels used in these papers, to control results derived from assumptions were, firstly, they should reduce under some conditions to the results given by the four-vector space-time theory, and secondly to those of the Special Relativity Theory (SRT). Such theoretical hypotheses of time as a vector are presented as an extension of the four-dimension (4D) Minkowski space-time to six or more dimensions, the most of them consisting of the symmetrical 6D, three dimensions for time in the and three for space. Although

no experimental evidence supports these schemes, their mathematical consistency is a permanent motivation for a further discussion [7] [9]. D. Barwacz also undertakes the hypothesis of time as a vector in four-dimensions, the three spatial plus one temporal. He simplifies his task for studying them to only one spatial and one temporal, but his development is extensible to 4D [10].

Nevertheless, the present study takes a different line of presentation more general than that canonical used in LT in the sense that a stationary system and an inertial system moving along an inclined straight line are considered. The obtained results implied to work in Physics within a simple universe of three spatial dimensions (not 4D or 6D, or more). By this approach, without assumptions, we were able to derive a vectorial structure of time with spatial components t'_x, t'_y, t'_z , measured at O', in function of components t_x, t_y, t_z (measured by the observer located at the origin O of the stationary system). Thus, the deduced vector time obtained within these transformations is therefore a vector **depending** on spatial coordinates. It is also worth mentioning that Hongbao Ma briefly stated a vectorial presentation of time **depending** on spatial coordinates, without referring measurements to distinct inertial observers. Namely, by a very direct procedure he obtained the components of vector time, associated to a moving point, as the relation between the components of the radio-vector of the moving point and the magnitude of its velocity, i.e., $t_x = \frac{x}{v}$, $t_y = \frac{y}{v}$, $t_z = \frac{z}{v}$, [4].

It is also a crucial fact to keep in mind that the canonical and modern way of presenting the LT derivation (firstly focused in one-dimension, the X axis, and then extending, by the isotropy postulate, to the other coordinates) does not allow perceiving the vectorial features of time informed in this work.

A final statement condensing author's motivation for developing this work can be read in the (2004) work of Bernard Guy: "... *the relativity: This theory shows a general structure that is not uniquely linked to the properties of light: may be the photon will not have the last word...*".

"... *in a way, time a priori has a three dimensional content, and this point of view allows solving the problems arising in the standard relativity theory. **Standard relativity theory does work well only in one space dimension** with a simple duality between one space variable and one time variable. **In 3D, it does not work properly...**"[20].*

The current work consists of four sections and three annexes. In section 1, is shown a modern and simple way of deriving LT following an analogous procedure like that encountered in [21], preserving the statement structure of Lorentz [1]. In section 2, by means of examples the vectorial structure of time is revealed. Based on this fact LT is generalized to Vectorial Lorentz Transformations (VLT) where assumptions were needless. In Section 3 Local Lorentz Transformations were defined and developed consistently and Section 4 is devoted to the conclusions. In Annex 1 is presented three examples revealing clearly the time as a vector. In Annex 2 is shown the consistency of VLT with Maxwell Equations, and consequently is demonstrated the inconsistency of canonical LT with Maxwell Equations. In Annex 3 is shown how the VLT can be extended to curvilinear movement.

For text and formulas Microsoft Word and its Microsoft editor of equations 3.0 were used. Pictures were done using Microsoft PowerPoint.

2. Lorentz Transformations

A simple way modernly used to arrive at LT, under the postulate that velocity of light is constant and independent of the source in any inertial frame and that the laws of physics are the same in all inertial frames, is to consider two observers, with all the equipment for doing measurements of length, time and velocities onto moving projectiles; the first one located at the origin of coordinates of a fixed system O, and the second one located at the origin of coordinates of an inertial moving system O'. System O' moves at a constant velocity v , relative to O, such that X' and X axes are on the same line. The goal is to obtain relationships between the observer's measurements, such that they must be valid for any velocity of any projectile including that of photons, c . In order to arrive at a solution taking into account these conditions, the procedure starts by taking as projectile a pulse of light. As a control, resulting transformations should be consistent with Galilean transformations for inertial systems with relative velocity $v \ll c$.

When moving origin O' coincides with fixed origin O, at $t = t' = 0$, a pulse of light is sent parallel to X-axis, (**Fig. A1a, Annex 1**). Observers measure a component x of the light pulse displacement at O and a component x' at O'. (In this case the Galilean Transformations are: $x' = x - v.t$; $y' = y = 0$; $z' = z = 0$; $t' = t$).

By doing a Galilean reasoning, but using a scaling factor k , to be calculated, for mathematically preserving the constancy of the speed of light, c , it is established the following relationship for the X-axis component of the light pulse:

$$x' = k.(x - v.t) \quad (1)$$

Under these conditions, the light pulse must fulfill the following relationships:

$$x' = c.t' \quad \Rightarrow \quad t' = \frac{x'}{c} \quad x = c.t \quad \Rightarrow \quad t = \frac{x}{c} \quad (2a)$$

Or, under the same conditions, for any projectile traveling at velocity $u_x \leq c$:

$$x' = u'_x.t' \quad \Rightarrow \quad t' = \frac{x'}{u'_x}; \quad x = u_x.t \quad \Rightarrow \quad t = \frac{x}{u_x}; \quad u_x \neq u'_x \quad (2b)$$

Substituting relations from equations (2a) into equation (1), LT for time is obtained:

$$c.t' = k. \left(c.t - v.\frac{x}{c} \right) \quad \Rightarrow \quad t' = k. \left(t - \frac{v}{c^2}.x \right) \quad (3)$$

Now, change the observer's role, namely, start considering O' fixed and O, the moving system. Under this configuration it is clear that the observer at O' will see the system O moving at velocity $-v$, namely, going in opposite sense to the light pulse. Reasoning as in the original situation, a pulse of light is sent when O and O' coincide, and a similar

relation to that of (1) is constructed through the same constant k (see **Fig. A1b, Annex 1**). Thus, from O the following relationship is constructed:

$$x = k \cdot (x' + v \cdot t') \quad (4)$$

Because these equations must be valid in any situation, from equations (1) and (4), the following relationships hold:

$$\begin{aligned} x' &= k \cdot (x - v \cdot t) = k \cdot \left(x - \frac{v \cdot x}{c}\right) = k \cdot x \cdot \left(1 - \frac{v}{c}\right) \\ x &= k \cdot (x' + v \cdot t') = k \cdot \left(x' + \frac{v \cdot x'}{c}\right) = k \cdot x' \cdot \left(1 + \frac{v}{c}\right) \end{aligned}$$

Multiplying both equations, the value of factor k is obtained:

$$x' \cdot x = k^2 \cdot x \cdot x' \cdot \left(1 - \frac{v^2}{c^2}\right) \quad \Rightarrow \quad k = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \quad (5)$$

(For $v \ll c \Rightarrow k \cong 1$; $x' \cong x - v \cdot t$; $y' = y = 0$; $z' = z = 0$; $t' \cong t$. Namely, as it was expected, these transformations are reduced to those of Galileo).

In this way the set of transformation equations between both systems of coordinates are obtained. By extending the validity of these relationships to any movement of O' (see **Fig. A4, Annex 1**), “by the geometry of the problem” or by the Isotropy postulate, they become:

$$x' = \frac{x - v \cdot t}{\sqrt{1 - \frac{v^2}{c^2}}} \quad y' = y \quad z' = z \quad t' = \frac{t - \frac{v \cdot x}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}} = t \cdot \frac{1 - \frac{v}{c^2} \cdot u_x}{\sqrt{1 - \frac{v^2}{c^2}}} \quad (6)$$

Dividing by time, velocity transformations u of light pulse are,

$$u'_x = \frac{u_x - v}{1 - \frac{v \cdot u_x}{c^2}} \quad u'_y = \frac{u_y \cdot \sqrt{1 - \frac{v^2}{c^2}}}{1 - \frac{v \cdot u_x}{c^2}} \quad u'_z = \frac{u_z \cdot \sqrt{1 - \frac{v^2}{c^2}}}{1 - \frac{v \cdot u_x}{c^2}} \quad (7)$$

First of all, it is worth mentioning that the second and third expressions in (6) do not come from equations or relationships but from *assumptions*. Again, the second and third expressions in (7) are direct consequence of such assumptions. It is important to emphasize that LT in (6) and (7) are currently accepted all over the world since the Special Theory of Relativity arrived at, a century ago.

One of the aspects that disenchant to those trying to see more inside LT is that it is impossible to find their derivation in a generalized form to any type of movement of the inertial system O'. In fact, as far as I know, in all the publications taking up this subject, the moving system O' is always confined to move on the X axis, extrapolating by “common sense” or any other postulate, to the general movement of O' [1], [2], [3], [8], [10], [16]; ¿Why? ¿Why cannot the system O' be generally presented as moving along an inclined trajectory, not coinciding with any axis? It can be understood that such configuration introduces problems for establishing the previously mentioned

assumptions, -until now untouchables and accepted-. In this work is concluded that such assumptions are unnecessary and hence, groundless. **in this work, as expected, it is presented a demonstration that LT are mathematically inconsistent with Maxwell Equations, see Annex 2.**

Obviously, Lorentz objective was to correct Galilean transformations in order to obtain general transformations that preserve the constancy of the light speed and also to be consistent with Maxwell equations so that they remained the same in any inertial system. In this work, LT are generalized by taking the moving system O' to move on an inclined line. This configuration allowed us to discover new features about time and took us to develop LT simply and consistently.

3. Is Time A Vector?

We are going to check that the answer to this question is yes. In this section we will depict the way how we were forced to arrive at the following concept of time: “Time is not only different for observers with distinct inertial movements, but additionally, it behaves between them as a vector”. As it will be observed, this concept will not result from any assumption. Instead, the vector structure will be deduced from the analysis of time's obtained expressions, for one, two, three (or more, if it were necessary) spatial dimensions. The two-dimensional case presented in **Annex 1, Fig. A3**, is a very illustrative example to clarify this concept, and it is repeated here in more detail in the next paragraphs:

Referring to figure A3, when O', moving along an inclined line, and O coincide, a light pulse is sent in any direction. By defining α , as the angle between trajectory of O' and X axis, and reasoning as in Part 1, the following equations hold:

$$\begin{aligned} x^2 + y^2 = c^2.t^2 & \quad x' = k.(x - v.t.\cos\alpha) \\ & \text{for,} \\ x'^2 + y'^2 = c^2.t'^2 & \quad y' = k.(y - v.t.\sin\alpha) \end{aligned}$$

Based on these previous relationships, by substituting, working on and grouping properly, we obtain:

$$\begin{aligned} c^2.t'^2 = x'^2 + y'^2 &= k^2.[(x - v.t.\cos\alpha)^2 + (y - v.t.\sin\alpha)^2] \\ c^2.t'^2 &= k^2.[(x^2 + y^2) + [v^2.(t.\cos\alpha)^2 + v^2.(t.\sin\alpha)^2] - 2.v.x.(t.\cos\alpha) - 2.v.y.(t.\sin\alpha)] \\ c^2.t'^2 &= k^2.[c^2.(t)^2 + v^2.(t)^2 - 2.v.[x.(t.\sin\alpha) + y.(t.\sin\alpha)]] \end{aligned}$$

Substituting: $c^2.t'^2 \equiv c^2.t'^2.(sin^2\alpha + cos^2\alpha)$ and $v^2.t'^2 = v^2.\frac{x^2+y^2}{c^2}$, we get:

$$\begin{aligned} c^2.t'^2 &= k^2.\{[c^2.(t.\cos\alpha)^2 - 2.v.x.(t.\cos\alpha) + v^2.\frac{x^2}{c^2}] + [c^2.(t.\sin\alpha)^2 + 2.v.y.(t.\sin\alpha) + v^2.\frac{y^2}{c^2}]\} \\ c^2.t'^2 &= k^2.[(c.t.\cos\alpha - \frac{v}{c}.x)^2 + (c.t.\sin\alpha - \frac{v}{c}.y)^2] \end{aligned}$$

From the last relationship, it is obtained the following expression for time:

$$t'^2 = k^2.[(t.\cos\alpha - \frac{v}{c^2}.x)^2 + (t.\sin\alpha - \frac{v}{c^2}.y)^2]$$

By observing carefully the right hand side of the previous expression, it reminds us the module of a vector. Thus, as it is suggested, the previous modular expression can be re-organized into its corresponding two-dimensional vectorial structure, in the following way:

$$\mathbf{t}' = k. \left[\left(t. \cos \alpha - \frac{v}{c^2}.x \right) \mathbf{i} + \left(t. \sin \alpha - \frac{v}{c^2}.y \right) \mathbf{j} \right] = k. \left[t. \cos \alpha. \mathbf{i} - \frac{v}{c^2}.x. \mathbf{i} + t. \sin \alpha. \mathbf{j} - \frac{v}{c^2}.y. \mathbf{j} \right]$$

$$\mathbf{t}' = k. \left[\left(t. \cos \alpha. \mathbf{i} + t. \sin \alpha. \mathbf{j} \right) - \frac{v}{c^2}. (x. \mathbf{i} + y. \mathbf{j}) \right] = k. \left[(t_x. \mathbf{i} + t_y. \mathbf{j}) - \frac{v}{c^2}. (x. \mathbf{i} + y. \mathbf{j}) \right]$$

Thus, by defining:

$$\left\{ \begin{array}{l} t_x = t. \cos \alpha \\ t_y = t. \sin \alpha \\ \mathbf{t} = t_x \mathbf{i} + t_y \mathbf{j} \end{array} \right\} \text{ and } \left\{ \begin{array}{l} t'_x = k. \left(t_x - \frac{v}{c^2}.x \right) \\ t'_y = k. \left(t_y - \frac{v}{c^2}.y \right) \end{array} \right\} \Rightarrow \left\{ \begin{array}{l} \mathbf{t}' = k. \left(\mathbf{t} - \frac{v}{c^2}.\mathbf{r} \right) \\ \mathbf{r}' = k. \left(\mathbf{r} - v.\mathbf{t} \right) \end{array} \right\}$$

It can be realized that this vector structure of time can be easily obtained for any number of dimensions by repeating this same procedure (**see Annex 1**). So, the vector character of time is not the result of any hypothesis; it comes directly from observing vector properties clearly present inside transformations relating measurements of both inertial observers. It can also be observed that, from an epistemological point of view, time as vector forms its direction by taking it from the vector velocity v of the moving system O' , leaving such parameter with a scalar character and functioning as a scaling factor. This can be understood due to both observers are on the same inclined line, which will imply the scalar character of v . Another epistemological characteristic of vector time is its dependence on coordinates x , y and z , which means that **it is not an independent vector**, a characteristic that appears as remarkable because differs to that of four dimensions (time as a fourth independent dimension) introduced by Einstein. This also means that we are still working in three spatial dimensions in this study, and that magnitudes can continue being defined as in classical physics, but with a modern and relativistic view. From observing these **results** we can develop the following definition of time: Time is forced to behave as a vector with spatial components in each coordinate, when it appears inside the VLT, but it can appear behaving as a scalar when it is not an element of a transformation such as VLT in the way we always have known it: as a sequential meter of events. But moreover, time can also be considered as a vector in the natural way it was referred to previously in [4]. In accordance with this idea and it perfectly applies to our work, Hongbao Ma says: *“this three dimensional time concept is obtained from the mathematical conception rather than the ontological existence. Mathematical results are at the epistemological level”* [4].

By considering time as having the properties of a vector when reflected within the relation between inertial observers with different movements, let's formally obtain the vectorial version for the Lorentz transformations (VLT). So, now we will refer in general to the three-dimensional case, or for further research it could be thought in an n-dimensional case, (**see Annex 1, Fig. A5**), where system O' moves on an inclined line and both observers will measure the light pulse radio-vectors \mathbf{r} , \mathbf{r}' . Vectors from now on will be written as boldface letters. The relationships in VLT, previously seen, are easily obtained

from Fig. A5:

$$\mathbf{r} = c.\mathbf{t} \quad \mathbf{r}' = c.\mathbf{t}' \quad \mathbf{t}' = \frac{\mathbf{r}'}{c} \quad \mathbf{t} = \frac{\mathbf{r}}{c}$$

$$\left\{ \begin{array}{l} \mathbf{r}' = k(\mathbf{r} - v.\mathbf{t}) \Rightarrow c.\mathbf{t}' = k.\mathbf{t}.(c - v) \\ \mathbf{r} = k(\mathbf{r}' + v.\mathbf{t}') \Rightarrow c.\mathbf{t} = k.\mathbf{t}'.(c + v) \end{array} \right\} \Rightarrow c^2.\mathbf{t}.\mathbf{t}' = k^2.\mathbf{t}'.\mathbf{t}.(c^2 - v^2) \Rightarrow k^2 = \frac{1}{1 - \frac{v^2}{c^2}}$$

$$\mathbf{r}' = \frac{\mathbf{r} - v.\mathbf{t}}{\sqrt{1 - \frac{v^2}{c^2}}} \Rightarrow c.\mathbf{t}' = \frac{c.\mathbf{t} - v.\frac{\mathbf{r}}{c}}{\sqrt{1 - \frac{v^2}{c^2}}} \Rightarrow \mathbf{t}' = \frac{\mathbf{t} - \frac{v}{c^2}.\mathbf{r}}{\sqrt{1 - \frac{v^2}{c^2}}} \Rightarrow \mathbf{u}' = \frac{d\mathbf{r}'}{dt'} = \frac{d\mathbf{r} - v.d\mathbf{t}}{\left| dt - \frac{v}{c^2}.d\mathbf{r} \right|} \quad (8)$$

The following equality also holds as invariant for VLT: $c^2.t'^2 - r'^2 = c^2.t^2 - r^2$. This means that the cinematic VLT, composed by expressions, \mathbf{r}' , \mathbf{t}' and \mathbf{u}' in (8), are generally valid for a light pulse or for any projectile moving at any speed less than c . As a check, the Jacobian matrix for any value of variables, becomes symmetric and equal to one, i.e., Letting x^i , be the variables measured by O, and \bar{x}^j , be those measured by O', for $i, j = 1, 2, 3$, and for $k = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$, we have:

$$\begin{array}{l} \mathbf{r}' = k.(\mathbf{r} - v.\mathbf{t}) \quad \mathbf{r} = k.(\mathbf{r}' + v.\mathbf{t}') \\ \mathbf{t}' = k.(\mathbf{t} - \frac{v}{c^2}.\mathbf{r}) \quad \mathbf{t} = k.(\mathbf{t}' + \frac{v}{c^2}.\mathbf{r}') \end{array} \Rightarrow \begin{array}{l} \left(\frac{\partial x^i}{\partial \bar{x}^j} \right) = \begin{bmatrix} k & -k.v \\ -k.\frac{v}{c^2} & k \end{bmatrix} \\ \left(\frac{\partial \bar{x}^j}{\partial x^i} \right) = \begin{bmatrix} k & +k.v \\ +k.\frac{v}{c^2} & k \end{bmatrix} \end{array} \quad \left(\frac{\partial x^i}{\partial \bar{x}^j} \right) = \left(\frac{\partial \bar{x}^j}{\partial x^i} \right) = 1$$

This is valid for any set of components: $\left\{ \begin{array}{l} y' = k.(y - v.t_y) \quad y = k.(y' + v.t'_y) \\ t'_y = k.(t_y - \frac{v}{c^2}.y) \quad t_y = k.(t'_y + \frac{v}{c^2}.y') \end{array} \right\}$

A final remark on the procedure previously presented: needless to say is that it was not done any assumption for obtaining the VLT presented in (8). Thus, because these are vectorial relationships, they are generally valid for any number of dimensions. It is opportune to say that the consistency of VLT with Maxwell Equations is demonstrated In Annex 2.

Let's obtain the expressions for VLT in three dimensions using spherical coordinates (**Fig. A5**). Allow β to be the angle between the inclined trajectory of O' and the plane XY; and allow α to be the angle formed by the projection of the inclined trajectory of O' on the plane XY, with the X-axis. When moving origin O' and fixed one O coincide, the light pulse is sent towards the space with generic components x, y, z , The general VLT of the vector time and that of the radio-vector of the pulse of light (or projectile), in three

dimensions, become:

$$\begin{aligned} x' &= \frac{x-v.t.\cos\alpha.\cos\beta}{\sqrt{1-\frac{v^2}{c^2}}} & t_x &= t.\cos\beta.\cos\alpha & x' &= \frac{x-v.t_x}{\sqrt{1-\frac{v^2}{c^2}}} \\ y' &= \frac{y-v.t.\sin\alpha.\cos\beta}{\sqrt{1-\frac{v^2}{c^2}}} & t_y &= t.\cos\beta.\sin\alpha & y' &= \frac{y-v.t_y}{\sqrt{1-\frac{v^2}{c^2}}} \\ z' &= \frac{z-v.t.\sin\beta}{\sqrt{1-\frac{v^2}{c^2}}} & t_z &= t.\sin\beta & z' &= \frac{z-v.t_z}{\sqrt{1-\frac{v^2}{c^2}}} \\ t' &= \left| \frac{\mathbf{t} - \frac{v}{c^2} \cdot \mathbf{r}}{\sqrt{1-\frac{v^2}{c^2}}} \right| = \sqrt{\frac{(t_x - \frac{v}{c^2} \cdot x)^2 + (t_y - \frac{v}{c^2} \cdot y)^2 + (t_z - \frac{v}{c^2} \cdot z)^2}{1-\frac{v^2}{c^2}}} \end{aligned}$$

The expressions for the velocities of the pulse of light or any projectile are obtained from the previous ones:

$$\begin{aligned} u'_x &= \frac{u_x - v.\cos\alpha.\cos\beta}{\sqrt{(\cos\alpha.\cos\beta - \frac{v.u_x}{c^2})^2 + (\sin\alpha.\cos\beta - \frac{v.u_y}{c^2})^2 + (\sin\beta - \frac{v.u_z}{c^2})^2}} \\ u'_y &= \frac{u_y - v.\sin\alpha.\cos\beta}{\sqrt{(\cos\alpha.\cos\beta - \frac{v.u_x}{c^2})^2 + (\sin\alpha.\cos\beta - \frac{v.u_y}{c^2})^2 + (\sin\beta - \frac{v.u_z}{c^2})^2}} \\ u'_z &= \frac{u_z - v.\sin\beta}{\sqrt{(\cos\alpha.\cos\beta - \frac{v.u_x}{c^2})^2 + (\sin\alpha.\cos\beta - \frac{v.u_y}{c^2})^2 + (\sin\beta - \frac{v.u_z}{c^2})^2}} \end{aligned} \quad (9)$$

Now, let's particularize these general results to the conditions from where the original LT were obtained (**Fig. A4**). If we re-establish such conditions (the system O' moving along the X axis, and the light pulse sent to space), i.e., for $\alpha = \beta = 0$ we will obtain the VLT version of the original Lorentz transformations:

$$\begin{aligned} x' &= \frac{x-v.t}{\sqrt{1-\frac{v^2}{c^2}}} & t_x &= t \\ y' &= \frac{y}{\sqrt{1-\frac{v^2}{c^2}}} & t_y &= 0 \\ z' &= \frac{z}{\sqrt{1-\frac{v^2}{c^2}}} & t_z &= 0 \end{aligned} \quad t' = \sqrt{\frac{(t_x - \frac{v}{c^2} \cdot x)^2 + (\frac{v}{c^2} \cdot y)^2 + (\frac{v}{c^2} \cdot z)^2}{1-\frac{v^2}{c^2}}} \quad (10)$$

$$\begin{aligned} u'_x &= \frac{u_x - v}{\sqrt{(1-\frac{v.u_x}{c^2})^2 + (\frac{v.u_y}{c^2})^2 + (\frac{v.u_z}{c^2})^2}} & u'_y &= \frac{u_y}{\sqrt{(1-\frac{v.u_x}{c^2})^2 + (\frac{v.u_y}{c^2})^2 + (\frac{v.u_z}{c^2})^2}} \\ u'_z &= \frac{u_z}{\sqrt{(1-\frac{v.u_x}{c^2})^2 + (\frac{v.u_y}{c^2})^2 + (\frac{v.u_z}{c^2})^2}} & u_x'^2 + u_y'^2 + u_z'^2 &= u_x^2 + u_y^2 + u_z^2 = c^2 \end{aligned} \quad (11)$$

Let's check the last relationship in (11), which is valid only for photons. In such equation is then implied that the velocity of light measured by any of the two observers should be the same, c (In general for any other projectile, $u'^2 \neq u^2$), i. e., on the basis of which O measures, $u_x^2 + u_y^2 + u_z^2 = c^2$, then O' will measure:

$$\begin{aligned} u_x'^2 + u_y'^2 + u_z'^2 &= \frac{(u_x - v)^2 + u_y^2 + u_z^2}{(1-\frac{v.u_x}{c^2})^2 + (\frac{v.u_y}{c^2})^2 + (\frac{v.u_z}{c^2})^2} = \frac{u_x^2 - 2.v.u_x + v^2 + u_y^2 + u_z^2}{1-\frac{2.v.u_x}{c^2} + \frac{v^2.u_x^2}{c^4} + \frac{v^2.u_y^2}{c^4} + \frac{v^2.u_z^2}{c^4}} \\ &= \frac{(u_x^2 + u_y^2 + u_z^2) - 2.v.u_x + v^2}{1-\frac{2.v.u_x}{c^2} + \frac{v^2}{c^4} \cdot (u_x^2 + u_y^2 + u_z^2)} = \frac{c^2 - 2.v.u_x + v^2}{1-\frac{2.v.u_x}{c^2} + \frac{v^2}{c^2}} = \frac{c^2 \cdot (1-\frac{2.v.u_x}{c^2} + \frac{v^2}{c^2})}{1-\frac{2.v.u_x}{c^2} + \frac{v^2}{c^2}} = c^2 \end{aligned}$$

When comparing equations (10) and (11), with the original LT equations (6) and (7), the first thing we realize is that components y , z , measured by the fixed observer are different to those of y' , z' , measured by the moving observer, thus, contradicting LT's statements. Additionally, we can observe that the expression of time is completely different to that of Lorentz in (6). And of course, the obtained expressions, according to this work, for velocity components u'_x , u'_y , u'_z , are also different of those presented for LT in (7). By the way, recently J. H. Field following the canonical form of presentation of the LT in a detailed manner, shows as mathematically correct the assumptions $y' = y$ and $z' = z$ [19]. On the contrary, according to the current work these assumptions were shown to be groundless. Thus, in author's opinion, there are only two relevant possibilities for obtaining such disagreement: either isotropy postulate is not applicable for this configuration or postulate can't be applied in relativity. Further research will answer this question.

Given that our procedure to obtain the vectorial transformations did not use any type of assumptions, it sufficiently demonstrates that in LT they were needless and therefore, LT canonical procedure is reduced to be only valid for one spatial dimension. Thus, its validity cannot be extrapolated to a general configuration.

It is conceivable that when in 1905 Einstein established his remarkable concept of the variation of mass with its velocity [2], he was actually looking for the one-to-one variation of physical magnitudes between classic and relativistic physics through Lorentz factors. At that time he already "had" the relationships for length, time, velocity and mass. So, Einstein probably got to consider Lorentz transformations (*not general, as we have shown previously*) as a central part of the SRT [3]. In an author's speculative opinion, Einstein later abandons SRT due to some observed inconsistencies and limitations of LT, and may be this was one of reasons he had for developing the General Theory of Relativity (GRT) trying to avoid such type of limitations in his research.

4. Local Lorentz Transformations

As it is commonly expressed in relativistic literature, the practical consequences of LT, under conditions of simultaneity of events, or their occurrence at the same location, are those known as length contraction and time dilation, respectively.

The simultaneity of events and occurrence at the same place can be reduced to the situation of two observers measuring the same physical magnitude from the same reference. In author's opinion, Einstein probably also tried to reach this result, but he was not successful due to the observed limitations of the LT.

The question can be posed again, now under the VLT: What are those consequences of the VLT that affect in a practical way our life? And how can we manage these practical consequences?

For example, when a physicist conceives that a pulse of light lasts eight and a half minutes coming from the Sun to the Earth, and he receives such image in his eyes, he establishes that Sun is not there but 15.300 KM far apart that place. From this point of view, he is thinking in a way that instantaneously reflects a real view of the universe.

Images are never real but thoughts like the previous ones are instantaneous. In order to systematize these ideas and to apply them in a real comparison of measurements, let's establish the following two conventions:

(1) From now on, both observers will do their measurements taking the same reference.

Let the origin of the fixed observer be this reference. For example, if the fixed observer O measures the radio-vector \mathbf{r} which corresponds to the displacement of a projectile sent to the space, the observer on the moving system O' will measure a similar radio-vector \mathbf{R}' from this same reference of the fixed observer, such that \mathbf{R}' will fulfill $\mathbf{r}' = \mathbf{R}' - \mathbf{R}_0^|$, with the definitions of the variables \mathbf{R}' and $\mathbf{R}_0^|$ given below:

$$\mathbf{R}' = \frac{\mathbf{r}}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{\mathbf{r} - v \cdot \mathbf{t}}{\sqrt{1 - \frac{v^2}{c^2}}} + \frac{v \cdot \mathbf{t}}{\sqrt{1 - \frac{v^2}{c^2}}} = \mathbf{r}' + \mathbf{R}_0^| \quad \text{for} \quad \mathbf{R}_0^| = \frac{v \cdot \mathbf{t}}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{\mathbf{r}_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

(1) The moving observer O' will not send any pulse of light (or projectile). Thus, he will measure a null displacement of the projectile. Then the radio-vector of his moving system, \mathbf{r}_0 , will be the only measurement completed. Thus, from (8) the expressions of the Local Lorentz Transformations (LLT) are obtained:

$$\mathbf{r}' = 0 \Rightarrow \mathbf{r} = v \cdot \mathbf{t} = \mathbf{r}_0; \quad \mathbf{t}' = \frac{\mathbf{t} - \frac{v}{c^2} \cdot v \cdot \mathbf{t}}{\sqrt{1 - \frac{v^2}{c^2}}} \Rightarrow \mathbf{t}' = \mathbf{t} \cdot \sqrt{1 - \frac{v^2}{c^2}}; \quad \mathbf{R}' = \frac{\mathbf{r}}{\sqrt{1 - \frac{v^2}{c^2}}} \quad (12)$$

In sum, the moving and fixed observers are referring their measurements to the “same time” and “same radio-vector” of system O'. As we observe in (12), time and distance vectors are related by a characteristic-scaling factor. The scaling factor (< 1) corresponding to time is a multiplier in any dimension or coordinate; for the case of distances it is a divider. In other words, a sphere expands uniformly in all directions.

However, It is important to point out that for the case of distances, the LLT are referred to the length measurements of the radio-vector that aims towards O', made by both observers **relative to the origin O**. This is completely different to what is done for VLT, where each observer does measurements relative to its own reference system. Namely, the transformations referred to LLT are different to those of VLT.

With those two conventions in mind, we will not be worried about location coincidence or simultaneity of events. The relations (12) imply that in LLT each physical magnitude, by virtue of its dependency on velocity, in a true way either contract, expand, growth or reduce, with the same scaling factor in all dimensions, independently of their image or how we see them. For example, if on the moving system the observer at O' measures a bar of length L_0 , lasting a time t_0 in his measuring, the observer on fixed system at O will measure this length as L and “time t_0 ” as t . The position of the bar in system O' is not relevant; what is important is that it is at rest for the observer at O' and moving on relative to O. Thus, the relationship between both measurements, according to LLT, will be:

$$L_0 = \frac{L}{\sqrt{1 - \frac{v^2}{c^2}}}; \quad t_0 = t \cdot \sqrt{1 - \frac{v^2}{c^2}} \quad \Rightarrow \quad L = L_0 \cdot \sqrt{1 - \frac{v^2}{c^2}}; \quad t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}} \quad (13)$$

This indicates that an observer in a “stationary” system O measures onto a moving bar at velocity v , a contraction from its original length L_0 to L , no matter which is the position of the bar in the system O', and a time dilation from t_0 to t , as it is shown in equations (13).

Lorentz factors in LLT act as scaling factors between measurements done at O and O', for any magnitude, no matter if this is a differential magnitude or an integral one. In other words, Lorentz factors are simply scaling factors between such measurements.

What is the real meaning of LLT expressed in equations (13)? First of all, **each component is affected by the Lorentz factor in the same way**, namely, contracting lengths. For example, If instead of a bar the observer in the moving system had had a squared bar, whose area, as we know, is the product of two lengths, then the obtained LLT of such area for observer at the fixed system, would be therefore the product of the two contracted lengths' LLT (later we will use this LLT characteristic of areas):

$$S' = S_0 = L_0^2 = \frac{L}{\sqrt{1 - \frac{v^2}{c^2}}} \cdot \frac{L}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{L^2}{1 - \frac{v^2}{c^2}} \Rightarrow S_0 = \frac{S}{1 - \frac{v^2}{c^2}} \Rightarrow S = S_0 \cdot \left(1 - \frac{v^2}{c^2}\right) \quad (14)$$

A volume $V' = L_{01} \cdot L_{02} \cdot L_{03}$, measured from O' is related to the volume measured from O, $V = L_1 \cdot L_2 \cdot L_3$, by the characteristic product of the three contracted lengths given below in (15):

$$V' = V_0 = L_{01} \cdot L_{02} \cdot L_{03} = \frac{L_1}{\sqrt{1 - \frac{v^2}{c^2}}} \cdot \frac{L_2}{\sqrt{1 - \frac{v^2}{c^2}}} \cdot \frac{L_3}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{L_1 L_2 L_3}{1 - \frac{v^2}{c^2}} \Rightarrow V_0 = \frac{V}{\left(1 - \frac{v^2}{c^2}\right)^{\frac{3}{2}}} \quad (15)$$

$$\Rightarrow V = V_0 \cdot \left(1 - \frac{v^2}{c^2}\right)^{\frac{3}{2}}$$

The velocity of the origin O' is obtained by differentiating the displacement of O' respect to time and substituting known LLTs. The LLT of velocity becomes:

$$\mathbf{v}' = \frac{d\mathbf{R}'}{dt'} = \frac{\frac{d\mathbf{r}_0}{\sqrt{1 - \frac{v^2}{c^2}}}}{dt \cdot \sqrt{1 - \frac{v^2}{c^2}}} = \frac{\frac{d\mathbf{r}_0}{dt}}{1 - \frac{v^2}{c^2}} \Rightarrow \mathbf{v}' = \frac{\mathbf{v}}{1 - \frac{v^2}{c^2}} \quad (16)$$

At this moment we realize that velocity of the moving system O', plays two roles: either as a scalar v , when it is inside the scaling factor, in where both observers see each other moving relative to themselves in the same line **under conditions of VLT**. Or, as a vector \mathbf{v}' , measured by the observer at O' into his own frame, but taking as reference the origin of the other system O, **under conventions of LLT**. It is important to be careful with these two different concepts!

After doing this necessary parenthesis, let's continue: LLT for acceleration is obtained in the same manner as in (16):

$$\mathbf{a}' = \frac{d\mathbf{v}'}{dt'} = \frac{\frac{d\mathbf{v}}{1 - \frac{v^2}{c^2}}}{dt \cdot \sqrt{1 - \frac{v^2}{c^2}}} = \frac{\frac{d\mathbf{v}}{dt}}{\left(1 - \frac{v^2}{c^2}\right)^{\frac{3}{2}}} \Rightarrow \mathbf{a}' = \frac{\mathbf{a}}{\left(1 - \frac{v^2}{c^2}\right)^{\frac{3}{2}}} \quad (17)$$

It is necessary to say at this moment that origin O' could be moving along an inertial curvilinear path following an inertial movement with variable velocity (**Annex 3, Figs. A6 and A7**). For example Earth has an undoubted inertial curvilinear movement around the Sun, and although it accelerates going to perihelion and reduce its speed after perihelion going to aphelion, we don't feel anything, buildings maintain their verticality, equilibrium of any kind is preserved, etc. So, with the transformations in equations (16) and (17), we would expect to obtain also LLT in Dynamics.

But first, let's obtain the transformation for angle between inertial systems. This magnitude emanate from the relation between curvilinear length of arc s and length of radius R . Because both magnitudes are lengths, Lorentz factors cancel out, and angle becomes invariant to LLT:

$$\alpha' = \frac{s'}{R'} = \frac{\frac{s}{\sqrt{1-\frac{v^2}{c^2}}}}{\frac{R}{\sqrt{1-\frac{v^2}{c^2}}}} = \frac{s}{R} \Rightarrow \alpha' = \alpha; \quad d\alpha' = \frac{ds'}{R'} = \frac{\frac{ds}{\sqrt{1-\frac{v^2}{c^2}}}}{\frac{R}{\sqrt{1-\frac{v^2}{c^2}}}} = \frac{ds}{R} \Rightarrow d\alpha' = d\alpha \quad (18)$$

In this way, angular velocity transforms as:

$$\omega' = \frac{d\alpha'}{dt'} = \frac{d\alpha}{dt \cdot \sqrt{1-\frac{v^2}{c^2}}} = \frac{\frac{d\alpha}{dt}}{\sqrt{1-\frac{v^2}{c^2}}} \Rightarrow \omega' = \frac{\omega}{\sqrt{1-\frac{v^2}{c^2}}} \quad (19)$$

Let's try to obtain a dynamical transformation for Force, based on already known LLT of magnitudes. Let's suppose two masses rotating circularly around a center of mass C at the same angular velocity ω . See Figure below:

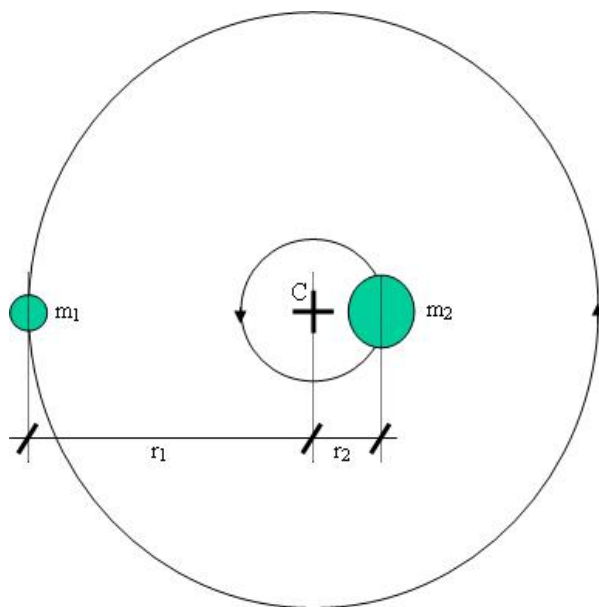


Fig. 1 Two masses rotating around a fixed center C

Because we have forced the masses to describe circular paths, it will allow us to do the following equivalent model to take over gravitational forces, i.e., only centrifugal forces

will be considered. Suppose a Hercules, located at the center of mass C, fixed, sustaining each mass through strong cords with each arm. Let there be three observers: Hercules at C, observer 1 on mass m_1 at a cord-distance r_1 from C, and observer 2 on mass m_2 at a cord-distance r_2 from C.

- (1) As a first conclusion, for Hercules to be in equilibrium, he must measure equal and opposite tensions in each arm. Thus: $m_1 \cdot \omega^2 \cdot r_1 = m_2 \cdot \omega^2 \cdot r_2$.
- (2) The tension T_1 exerted at one of Hercules' arm by cord r_1 , measured by observer 1 on m_1 , will be $m'_1 \cdot \omega'^2 \cdot r'_1$, and tension T_2 exerted at Hercules' other arm by the cord r_2 , measured by observer 2 on m_2 , will be $m''_2 \cdot \omega''^2 \cdot r''_2$. Let's assume that tensions T_1 and T_2 are equal, in order to maintain, as before, Hercules in equilibrium, which would also mean that Force should be invariant to LLT. The magnitudes involved in both tensions are transformed with respect to what is measured by Hercules, the fixed observer, in the following manner (except for masses, whose transformation is unknown):

$$m'_1 \cdot \omega'^2 \cdot r'_1 = m'_1 \cdot \frac{\omega^2}{\left(1 - \frac{v_1^2}{c^2}\right)} \cdot \frac{r_1}{\sqrt{1 - \frac{v_1^2}{c^2}}} \equiv m''_2 \cdot \omega''^2 \cdot r''_2 = m''_2 \cdot \frac{\omega^2}{\left(1 - \frac{v_2^2}{c^2}\right)} \cdot \frac{r_2}{\sqrt{1 - \frac{v_2^2}{c^2}}}$$

The only way for this relationship to always be consistent for any values of v_1 and v_2 is that masses have the following LLT:

$$m'_1 = \left(1 - \frac{v_1^2}{c^2}\right)^{\frac{3}{2}} \cdot m_1 \quad \text{and} \quad m''_2 = \left(1 - \frac{v_2^2}{c^2}\right)^{\frac{3}{2}} \cdot m_2 \quad (20)$$

In this way Lorentz factors cancel out and this would imply: $m_1 \cdot \omega^2 \cdot r_1 = m_2 \cdot \omega^2 \cdot r_2$, But, as this equality was previously correctly concluded in 1), then our assumption is also correct. This can be seen in another way. For maintaining Hercules in equilibrium (first conclusion), then tensions T_1 and T_2 must be equal. Thus, these results lead to both statements imply each other, i.e.,

$$T_1 = m'_1 \cdot \omega'^2 \cdot r'_1 \equiv m_1 \cdot \omega^2 \cdot r_1 \equiv m_2 \cdot \omega^2 \cdot r_2 \equiv m''_2 \cdot \omega''^2 \cdot r''_2 = T_2.$$

Let's discuss in a deep way this equation. When observer 1 on m_1 (remember that he is fixed with respect to this mass, although the whole is a moving system) measures his mass, he measures m'_1 , which is, for him, the rest mass, $m'_1 = M_{01}$. The same applies for the other observer 2 measuring the mass where he is on: $m''_2 = M_{02}$. So, given that through this special case of circular movement we have obtained the Lorentz factors for such masses in (20), and because the LLT of a magnitude always has the same structure, we can conclude, from equation (20), with the following strong statement: **In general, an inertial mass in movement at a velocity v is related to its rest mass in the following manner:**

$$m = \frac{M_0}{\left(1 - \frac{v^2}{c^2}\right)^{\frac{3}{2}}} \quad (21)$$

This definition differs from the well-known Einstein's mass definition: $m = \frac{M_0}{\sqrt{1 - \frac{v^2}{c^2}}}$. In regards with this point, it's worth mentioning that Einstein also obtained equation (21)

in his remarkable paper of 1905. He called this mass “longitudinal mass” [2], but later he discarded it from his work.

Continuing the analysis by another route to obtain the equation (21), the following is a more general way to arrive at the same result. For instance, let’s consider Earth and Sun as if they were the only bodies of the inertial solar system. We are going to consider as if the sun was the fixed system, and Earth moving around the Sun. Thus, Angular Momentum of Earth under LLT, measured by an observer from the Sun, is $m.r^2.\omega$ and its value must be constant, because there are no more forces acting around, and conservation of angular momentum holds. The “same” Angular Momentum of Earth measured by another observer, on Earth, taking Sun as his reference for measurements, is $m'.r'^2.\omega'$, which must also be constant, because the laws of nature are the same in any system of coordinates. Let’s focus our attention on the explicit transformation of the elements involved within this last expression of angular momentum except for the earth mass, whose transformation is still considered unknown:

$$m'.r'^2.\omega' = m'.\frac{r^2}{\left(1 - \frac{v^2}{c^2}\right)} \cdot \frac{\omega}{\sqrt{1 - \frac{v^2}{c^2}}} = \text{CONSTANT} \quad (22)$$

By carefully observing equation (22), we conclude that the only way for this expression being always constant, for any value of the variable v present in Lorentz factors in the denominator, is that the transformation for $m' = M_0$, cancels out the effect of such factors. For instance:

$$m' = M_0 = \left(1 - \frac{v^2}{c^2}\right)^{\frac{3}{2}} \cdot m \quad \Rightarrow \quad m = \frac{M_0}{\left(1 - \frac{v^2}{c^2}\right)^{\frac{3}{2}}} \quad (23)$$

This is the same result previously obtained in equation (21). This also means that Angular Momentum is invariant under LLT (and also the force). Given that Local Lorentz factors equally influence any physical magnitude in all dimensions, we don’t have different LLT’s for the same magnitude, contrasting to which is found in the Special Theory of Relativity (remember longitudinal or transversal expressions of mass, fields, etc).

Given that LLT of mass is already known, let’s obtain other dynamical LLT.

Linear Momentum:

$$\mathbf{p}' = m'.\mathbf{v}' = m \cdot \left(1 - \frac{v^2}{c^2}\right)^{\frac{3}{2}} \cdot \frac{\mathbf{v}}{\left(1 - \frac{v^2}{c^2}\right)} \quad \Rightarrow \quad \mathbf{p}' = \sqrt{1 - \frac{v^2}{c^2}} \cdot \mathbf{p} \quad (24)$$

Observe this result: Linear Momentum is not invariant, as SRT states.

Angular Momentum:

$$\mathbf{L}' = \mathbf{r}' \times \mathbf{p}' = \frac{\mathbf{r}}{\sqrt{1 - \frac{v^2}{c^2}}} \times \sqrt{1 - \frac{v^2}{c^2}} \cdot \mathbf{p} \quad \Rightarrow \quad \mathbf{L}' = \mathbf{L}(\text{Invariant}) \quad (25)$$

Force:

$$\mathbf{F}' = \frac{d\mathbf{p}'}{dt'} = \frac{\sqrt{1 - \frac{v^2}{c^2}} \cdot d\mathbf{p}}{\sqrt{1 - \frac{v^2}{c^2}} \cdot dt} \Rightarrow \mathbf{F}' = \mathbf{F} \quad (\text{Invariant as expected}) \quad (26)$$

Kinetic Energy:

$$dE' = \mathbf{F}' \cdot d\mathbf{r}' = \mathbf{F} \cdot \frac{d\mathbf{r}}{\sqrt{1 - \frac{v^2}{c^2}}} \Rightarrow dE' = \frac{dE}{\sqrt{1 - \frac{v^2}{c^2}}} \quad (27)$$

Electromagnetic or Lorentz Force: (Must be invariant, because the magnitude of any force is invariant, see equation (26) and development of equation (20))

$$\mathbf{F}' = q' \cdot (\mathbf{\Xi}' + \mathbf{v}' \times \mathbf{B}')$$

Let's discuss this relationship. Electric Charge q seems not to be influenced by the velocity. Let's assume that it is invariant under LLT. Thus, in order to preserve the invariance of Force, Electric Field $\mathbf{\Xi}$ and the product $\mathbf{v} \times \mathbf{B}$ must be invariant. ***If this assumption is false, for sure a contradiction will arise later on.*** A good property of scaling factors in LLT is that they behave as if they had magnitude, allowing dimensional analysis based on characteristic Lorentz scaling factors. From the assumed LLT invariance of q , then $\mathbf{v}' \times \mathbf{B}'$ is also invariant:

Magnetic Field Density: Applying equation (16),

$$\mathbf{B}' = \mathbf{B} \cdot \left(1 - \frac{v^2}{c^2}\right) \quad (28)$$

Electric Field:

$$\mathbf{\Xi}' = \mathbf{\Xi} \quad (\text{Invariant}) \quad (29)$$

Electric Charge:

$$q' = q \quad (\text{Invariant}) \quad (30)$$

Electric Potential:

$$dV' = \mathbf{\Xi}' \cdot d\mathbf{r}' = \mathbf{\Xi} \cdot \frac{d\mathbf{r}}{\sqrt{1 - \frac{v^2}{c^2}}} \Rightarrow dV' = \frac{dV}{\sqrt{1 - \frac{v^2}{c^2}}} \quad (31)$$

Let's check the assumption for Electric Charge and its effects. Let's obtain the expression for the Electric Energy. It should lead to the already obtained expression (27) for energy. In fact:

$$dE' = q' \cdot dV' = q \cdot \frac{dV}{\sqrt{1 - \frac{v^2}{c^2}}} \Rightarrow dE' = \frac{dE}{\sqrt{1 - \frac{v^2}{c^2}}} \quad [\text{See equation (27)}]$$

Another check: An electric charge contained in a mass m located in an uniform magnetic field, which moves describing a circular path, should lead to the LLT of angular velocity, which is already known in equation (19),

$$\omega' = -q' \cdot \frac{\mathbf{B}'}{m'} = -q \cdot \frac{\mathbf{B} \cdot \left(1 - \frac{v^2}{c^2}\right)}{m \cdot \left(1 - \frac{v^2}{c^2}\right)^{\frac{3}{2}}} = \frac{-q \cdot \frac{\mathbf{B}}{m}}{\sqrt{1 - \frac{v^2}{c^2}}} \Rightarrow \omega' = \frac{\omega}{\sqrt{1 - \frac{v^2}{c^2}}} \text{ [see equation (19)]}$$

As it is seen, our assumption for charge is consistent. Furthermore, this control ratifies mass transformation. By continuing checking:

The Magnetic Field Density B on a point at a distance R from a current $I = \frac{dq}{dt}$, given by $B = \frac{\mu \cdot I}{2\pi \cdot R} \Rightarrow \mu = \frac{2\pi \cdot R \cdot B}{I}$, leads to obtain the transformation of the Magnetic Permeability:

$$\mu' = \frac{2\pi \cdot R' \cdot B'}{I'} \Rightarrow \mu' = \mu \cdot \left(1 - \frac{v^2}{c^2}\right) \quad (32)$$

Similarly, Electric Permittivity, ε , can be obtained from Gauss Law:

$$\oint_{S'} \boldsymbol{\Xi}' \cdot d\mathbf{S}' = \frac{q'}{\varepsilon'} \Rightarrow \oint_S \boldsymbol{\Xi} \cdot \frac{d\mathbf{S}}{\left(1 - \frac{v^2}{c^2}\right)} = \frac{q}{\varepsilon'} \Rightarrow \varepsilon' = \varepsilon \cdot \left(1 - \frac{v^2}{c^2}\right) \quad (33)$$

Electric Displacement:

$$\mathbf{D}' = \varepsilon' \cdot \boldsymbol{\Xi}' = \varepsilon \cdot \left(1 - \frac{v^2}{c^2}\right) \cdot \boldsymbol{\Xi} \Rightarrow \mathbf{D}' = \mathbf{D} \cdot \left(1 - \frac{v^2}{c^2}\right) \quad (34)$$

Current Density:

$$\mathbf{J}' = \frac{\frac{dq'}{dt'}}{S'} = \frac{\frac{dq}{dt} \cdot \sqrt{1 - \frac{v^2}{c^2}}}{\frac{S}{1 - \frac{v^2}{c^2}}} \Rightarrow \mathbf{J}' = \mathbf{J} \cdot \sqrt{1 - \frac{v^2}{c^2}} \quad (35)$$

Magnetic Field:

$$\mathbf{H}' = \frac{\mathbf{B}'}{\mu'} = \frac{\mathbf{B} \cdot \left(1 - \frac{v^2}{c^2}\right)}{\mu \cdot \left(1 - \frac{v^2}{c^2}\right)} \Rightarrow \mathbf{H}' = \mathbf{H} \text{ (Invariant)} \quad (36)$$

Magnetic Flux:

$$\phi' = \oint_{S'} \mathbf{B}' \cdot d\mathbf{S}' = \oint_{S'} \mathbf{B} \cdot \left(1 - \frac{v^2}{c^2}\right) \cdot \frac{d\mathbf{S}}{\left(1 - \frac{v^2}{c^2}\right)} \Rightarrow \phi' = \phi \text{ (Invariant)} \quad (37)$$

Checking:

$$\frac{\partial \boldsymbol{\Xi}'}{\partial r'} = -\frac{\partial \mathbf{B}'}{\partial t'} \Rightarrow \frac{\partial \boldsymbol{\Xi}}{\frac{\partial r}{\sqrt{1 - \frac{v^2}{c^2}}}} = -\frac{\partial \mathbf{B} \cdot \left(1 - \frac{v^2}{c^2}\right)}{\partial t \cdot \sqrt{1 - \frac{v^2}{c^2}}} \Rightarrow \frac{\partial \boldsymbol{\Xi}}{\partial r} = -\frac{\partial \mathbf{B}}{\partial t} \text{ [Checked]}$$

$$-\frac{\partial \mathbf{B}'}{\partial r'} = -\mu' \cdot \varepsilon \cdot \frac{\partial \boldsymbol{\Xi}'}{\partial t'} \Rightarrow \frac{\partial \mathbf{B} \cdot \left(1 - \frac{v^2}{c^2}\right)}{\frac{\partial r}{\sqrt{1 - \frac{v^2}{c^2}}}} = -\mu \cdot \varepsilon \cdot \left(1 - \frac{v^2}{c^2}\right)^2 \frac{\partial \boldsymbol{\Xi}}{\partial t \cdot \sqrt{1 - \frac{v^2}{c^2}}} \Rightarrow -\frac{\partial \mathbf{B}}{\partial r} = -\mu \cdot \varepsilon \cdot \frac{\partial \boldsymbol{\Xi}}{\partial t}$$

These results show that the relation between electric and magnetic fields holds in any reference system, as it was expected.

It can be shown that Maxwell Equations hold in any reference system under LLT. In fact, by taking into account that:

$$\nabla' = \frac{\partial}{\partial r'} = \frac{\partial}{\frac{\partial r}{\sqrt{1-\frac{v^2}{c^2}}}} \Rightarrow \nabla' = \sqrt{1-\frac{v^2}{c^2}} \cdot \nabla \quad (38)$$

$$1) \nabla' \times \mathbf{E}' = -\frac{\partial \mathbf{B}'}{\partial t'} \Rightarrow \sqrt{1-\frac{v^2}{c^2}} \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B} \cdot \frac{1-\frac{v^2}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}}}{\partial t \cdot \sqrt{1-\frac{v^2}{c^2}}} \Rightarrow \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$2) \nabla' \times \mathbf{H}' = \frac{\partial \mathbf{D}'}{\partial t'} + \mathbf{J}' \Rightarrow \sqrt{1-\frac{v^2}{c^2}} \nabla \times \mathbf{H} = \frac{\partial \mathbf{D} \cdot \frac{1-\frac{v^2}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}}}{\partial t \cdot \sqrt{1-\frac{v^2}{c^2}}} + \mathbf{J} \cdot \sqrt{1-\frac{v^2}{c^2}} \Rightarrow \nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t} + \mathbf{J}$$

$$3) \nabla' \cdot \mathbf{D}' = \rho' = \frac{q'}{V'} \Rightarrow \sqrt{1-\frac{v^2}{c^2}} \nabla \cdot \mathbf{D} \cdot \left(1-\frac{v^2}{c^2}\right) = \frac{q}{\left(1-\frac{v^2}{c^2}\right)^{\frac{3}{2}}} \Rightarrow \nabla \cdot \mathbf{D} = \rho$$

$$4) \nabla' \cdot \mathbf{B}' = 0 \Rightarrow \nabla \cdot \mathbf{B} \cdot \left(1-\frac{v^2}{c^2}\right) = 0 \Rightarrow \nabla \cdot \mathbf{B} = 0$$

Pointing Theorem. For

$$P' = \frac{dE'}{dt'} = \frac{\frac{dE}{\sqrt{1-\frac{v^2}{c^2}}}}{dt \cdot \sqrt{1-\frac{v^2}{c^2}}} = \frac{P}{\left(1-\frac{v^2}{c^2}\right)} \quad (39)$$

$$P' = Re \frac{1}{2} \oint \mathbf{E}' \times \mathbf{H}' \cdot d\mathbf{S}' \Rightarrow \frac{P}{\left(1-\frac{v^2}{c^2}\right)} = Re \frac{1}{2} \oint \mathbf{E} \times \mathbf{H} \cdot \frac{d\mathbf{S}}{\left(1-\frac{v^2}{c^2}\right)} \Rightarrow P = Re \frac{1}{2} \oint \mathbf{E} \times \mathbf{H} \cdot d\mathbf{S}$$

Observe the dimensional analysis' consistency of LLTs for each magnitude. All these consistent controls seem to confirm the correctness of the LLT approach.

It is necessary to remark that Lorentz factors are only scaling factors between measurements, no matter whether they are differentials or integrals. For instance if:

$$\mathbf{p}' = \sqrt{1-\frac{v^2}{c^2}} \cdot \mathbf{p} \quad \text{Then } d\mathbf{p}' = \sqrt{1-\frac{v^2}{c^2}} \cdot d\mathbf{p} \quad \text{or } \iiint d^3\mathbf{p}' = \iiint \sqrt{1-\frac{v^2}{c^2}} \cdot d^3\mathbf{p}$$

However, a different thing is the contraction suffered by a bar going through the space with velocity v , from a known length L_0 , to L , measured by one observer inside his own system (second observer does not exist), according to the law $L = L_0 \cdot \sqrt{1-\frac{v^2}{c^2}}$. This case is not a scaling one in the sense of the LLT, but a property of the bar whose **known length** depends on its velocity through the space for a stationary observer. For velocity v , variable, L is also variable. For this case the differential dL becomes, $dL = L_0 \cdot \frac{v \cdot dv}{1-\frac{v^2}{c^2}}$.

The same consideration must be made for a mass m that crosses the space with velocity v , with known rest mass m_0 . The expressions for the variable m , and its differential, depending on its velocity v , are:

$$m = \frac{M_0}{\left(1-\frac{v^2}{c^2}\right)^{\frac{3}{2}}} \Rightarrow dm = 3 \cdot M_0 \cdot c^3 \cdot \frac{v \cdot dv}{(c^2-v^2)^{\frac{5}{2}}} = 3 \cdot \frac{M_0 \cdot c^3}{(c^2-v^2)^{\frac{3}{2}}} \cdot \frac{v \cdot dv}{(c^2-v^2)} = 3 \cdot m \cdot \frac{v \cdot dv}{(c^2-v^2)}$$

As it should be noticed, it is important to be careful with such differences.

Another aspect to be emphasized is that of the vector character of time. This vectorial character is only noticed within the relationship between times measured by two inertial observers, through coordinate transformations, when a generalized configuration is used. Only under such condition, time is mathematically forced to appear as a vector. On the contrary, time measured by one observer in his own coordinate system (second observer does not exist) can behave as we are used to know it: as a scalar, although as it is encountered in the work done by Hongbao Ma, it is possible to express time in a vectorial form [4].

5. Conclusions

If Einstein's postulates are correct (in author's opinion they are), contradictions informed in this work for Lorentz Transformations (LT) will lead to a new field of research in theoretical physics that will bring new vigor to this science. Additionally, if the Local Lorentz Transformation (LLT), presented in this work, reveals itself to be a correct approach, it will bring to the surface a branch of physics that was neglected before being developed, as a transition between Classic, and Relativistic or Quantum Physics. In author's opinion, Einstein's work was intended to go in this sense, but unsolved contradictions, introduced by LT at the very starting point of his research, probably made Einstein leave in an abrupt manner the Special Theory of Relativity to lead his investigation into a more general development: the General Theory of Relativity. In the present study, we deliberately ignored Minkowski geometry and four-dimension space-time, because VLT allows doing space-time-varying analysis in three spatial dimensions, for any movement, rectilinear or curvilinear, respecting the constancy of light speed and maintaining the consistency with Maxwell Equations. Experimental validation of this approach, for example, the accuracy of the new definition of mass rigorously obtained in equation (23), will probably require complex experiments with known rest masses accelerated at speeds close to that of light in order to establish whether the value of mass is the well-known coined by Einstein or that of the equation (23).

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Annex 1

Time As A Vector. Examples

A) In one-dimensional space, one way to interpret this could be (See Fig. A1):

$$x = c.t \quad x' = k.(x - v.t)$$

$$t' = k.(t - \frac{v}{c^2}x) \quad \Rightarrow \quad \begin{aligned} \mathbf{t}' &= k.(\mathbf{t} - \frac{v}{c^2}.\mathbf{r}) \\ \mathbf{r}' &= k.(\mathbf{r} - v.\mathbf{t}) \end{aligned}$$

B) For a two-dimensional space, derivation is less direct, but simple. The following equations hold (See Fig. A3):

$$\begin{aligned} x^2 + y^2 &= c^2.t^2 & x' &= k.(x - v.t.\cos \alpha) \\ x'^2 + y'^2 &= c^2.t'^2 & y' &= k.(y - v.t.\sin \alpha) \end{aligned}$$

Where, α , is the angle between trajectory of O' and X axis; By defining time components as $t_x = t.\cos \alpha$, $t_y = t.\sin \alpha$, we obtain:

$$\begin{aligned} x' &= k.(x - v.t_x) \\ y' &= k.(t - v.t_y) \end{aligned} \quad \text{Substututing and grouping:}$$

$$\begin{aligned} c^2.t'^2 &= x'^2 + y'^2 = k^2.[(x - v.t_x)^2 + (y - v.t_y)^2] \\ c^2.t'^2 &= k^2.[x^2 + y^2 + v^2.t_x^2 + v^2.t_y^2 - 2.v.x.t_x - 2.v.y.t_y] \\ c^2.t'^2 &= k^2.[c^2.t^2 + v^2.t^2 - 2.v.(x.t_x + y.t_y)] \\ c^2.t'^2 &= k^2.[c^2.(t_x^2 + t_y^2) + v^2.\frac{x^2+y^2}{c^2} - 2.v.(x.t_x + y.t_y)] \\ c^2.t'^2 &= k^2.[(c.t_x - \frac{v}{c}.x)^2 + (c.t_y - \frac{v}{c}.y)^2] \end{aligned}$$

Dividing by c^2 , a vectorial structure of time is clearly obtained and follows:

$$t'^2 = k^2.[(t_x - \frac{v}{c^2}.x)^2 + (t_y - \frac{v}{c^2}.y)^2] \quad \Rightarrow \quad \begin{aligned} \mathbf{t}' &= k.(\mathbf{t} - \frac{v}{c^2}.\mathbf{r}) \\ \mathbf{r}' &= k.(\mathbf{r} - v.\mathbf{t}) \end{aligned}$$

C) For the three-dimensional case, see Fig. A5, the following relationships hold:

$$\begin{aligned} x^2 + y^2 + z^2 &= c^2.t^2 & x' &= k.(x - v.t.\cos \alpha.\cos \beta) & t_x &= t.\cos \alpha.\cos \beta \\ x'^2 + y'^2 + z'^2 &= c^2.t'^2 & y' &= k.(y - v.t.\sin \alpha.\cos \beta) & t_y &= t.\sin \alpha.\cos \beta \\ & & z' &= k.(z - v.t.\sin \beta) & t_z &= t.\sin \beta \end{aligned}$$

Following a similar procedure to that previously used is obtained again the familiar vector structure expression of time for three (or for any number of) dimensions:

$$t'^2 = k^2.[(t_x - \frac{v}{c^2}.x)^2 + (t_y - \frac{v}{c^2}.y)^2 + (t_z - \frac{v}{c^2}.z)^2] \quad \Rightarrow \quad \begin{aligned} \mathbf{t}' &= k.(\mathbf{t} - \frac{v}{c^2}.\mathbf{r}) \\ \mathbf{r}' &= k.(\mathbf{r} - v.\mathbf{t}) \end{aligned}$$

All these results lead consistently to consider the behavior of time as a vector when it is referred to observers located in systems with different inertial movements.

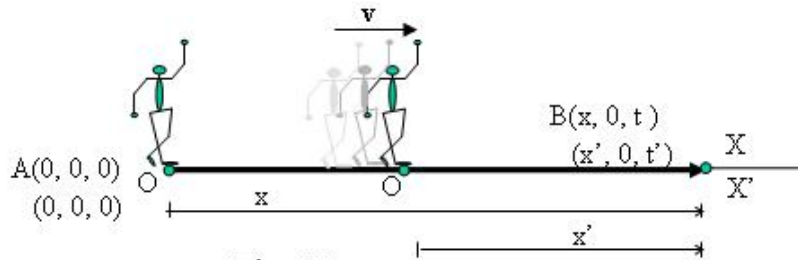


Fig. A1a

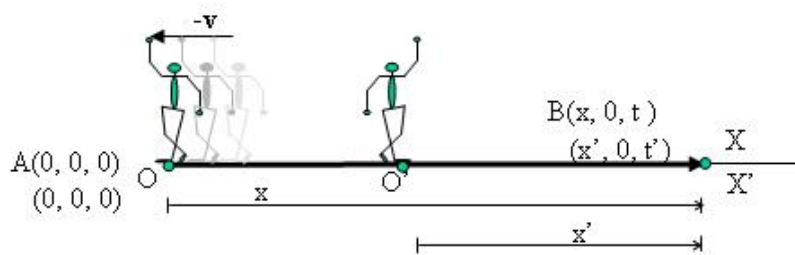


Fig. A1b

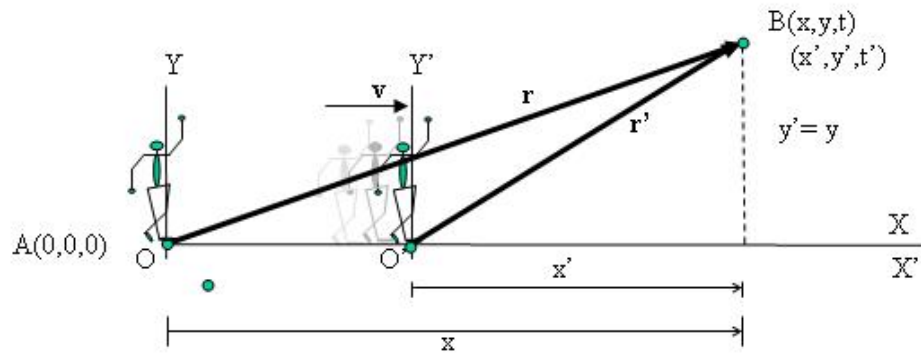


Fig. A2

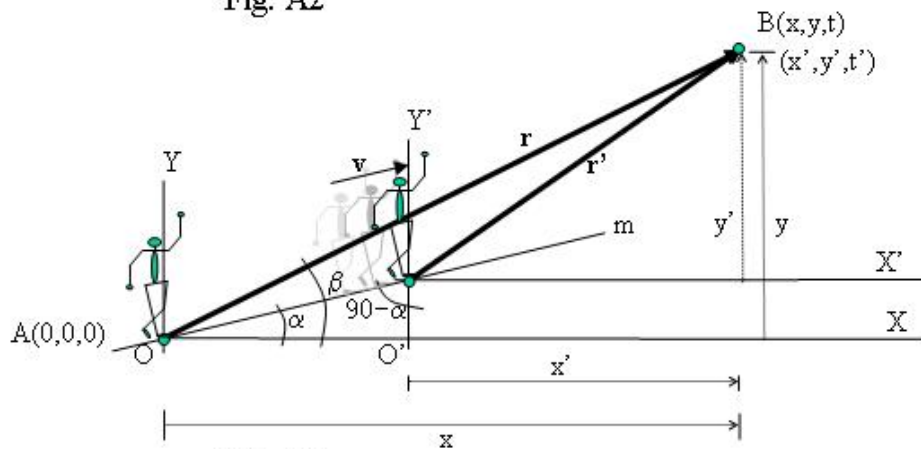


Fig. A3

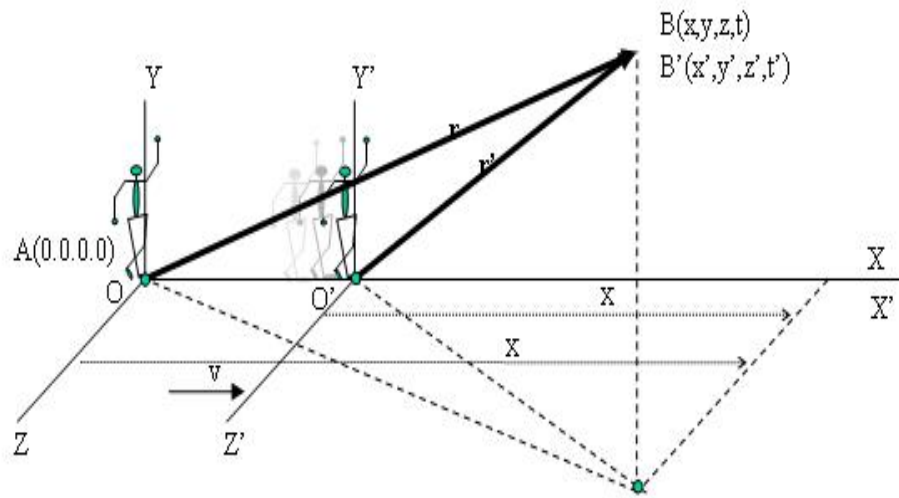


Fig. N° A4

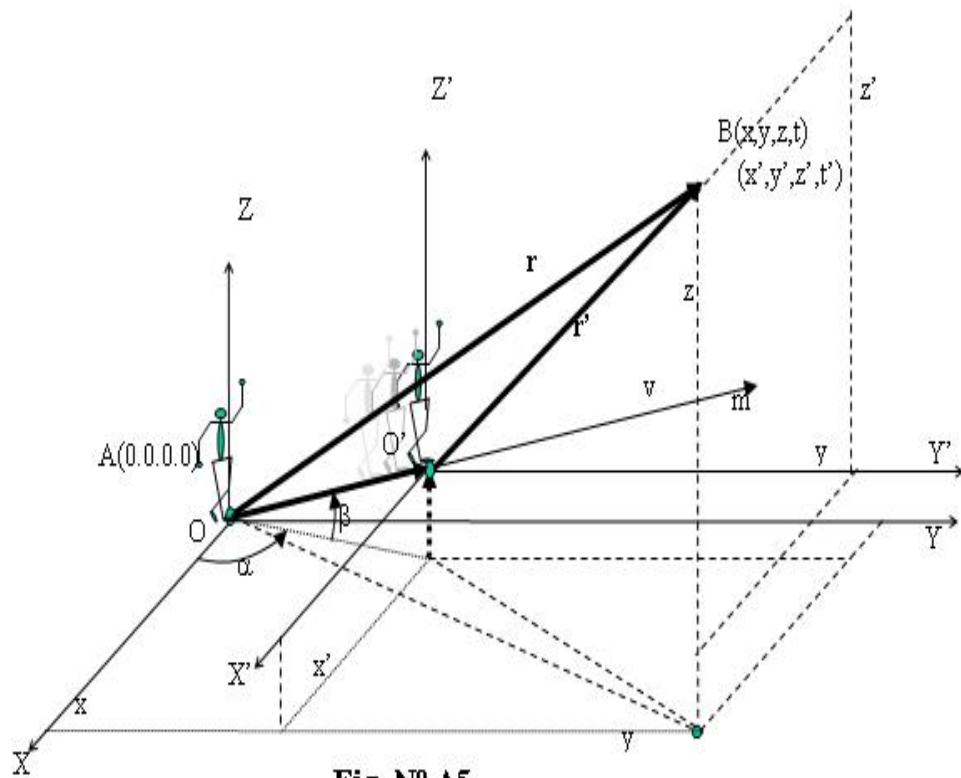


Fig. N° A5

Annex 2

Consistency Of Vectorial Lorentz Transformations And Inconsistency Of Lorentz Transformations

Given that we are working with vectors it is suitable to obtain a Wave Equation presentation in function of the light pulse radio-vector and time vector.

$$\mathbf{r} = x.\mathbf{i} + y.\mathbf{j} + z.\mathbf{k} \quad \Rightarrow \quad r = \sqrt{x^2 + y^2 + z^2} \quad \Rightarrow \quad \frac{\partial r}{\partial x} = \frac{x}{r}; \quad \frac{\partial r}{\partial y} = \frac{y}{r}; \quad \frac{\partial r}{\partial z} = \frac{z}{r};$$

In this way, each one of the components of the operator ∇ can be represented as depending on both vectors. Let's work to arrive at equations depending only on r and t . For this, we will develop the expressions of the components x , y , z :

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial r} \frac{\partial r}{\partial x} + \frac{\partial}{\partial t} \frac{\partial t}{\partial x}; \quad \frac{\partial}{\partial y} = \frac{\partial}{\partial r} \frac{\partial r}{\partial y} + \frac{\partial}{\partial t} \frac{\partial t}{\partial y}; \quad \frac{\partial}{\partial z} = \frac{\partial}{\partial r} \frac{\partial r}{\partial z} + \frac{\partial}{\partial t} \frac{\partial t}{\partial z}; \quad \text{Where, } \frac{\partial t}{\partial x} = \frac{\partial t}{\partial y} = \frac{\partial t}{\partial z} = 0$$

So, the operator $\nabla = \frac{\partial}{\partial x}\mathbf{i} + \frac{\partial}{\partial y}\mathbf{j} + \frac{\partial}{\partial z}\mathbf{k}$ can be expressed as:

$$\nabla = \frac{\partial}{\partial r} \frac{\partial r}{\partial x}\mathbf{i} + \frac{\partial}{\partial r} \frac{\partial r}{\partial y}\mathbf{j} + \frac{\partial}{\partial r} \frac{\partial r}{\partial z}\mathbf{k} = \frac{\partial}{\partial r} \frac{x}{r}\mathbf{i} + \frac{\partial}{\partial r} \frac{y}{r}\mathbf{j} + \frac{\partial}{\partial r} \frac{z}{r}\mathbf{k} = \frac{\partial}{\partial r} \frac{\mathbf{r}}{r} \quad \Rightarrow \quad \nabla \bullet \nabla = \nabla^2 = \frac{\partial^2}{\partial r^2}$$

Thus, Wave Equation can be put in a simpler manner, only as function of r and t :

$$\frac{\partial^2 \varepsilon}{\partial r^2} - \frac{1}{c^2} \cdot \frac{\partial^2 \varepsilon}{\partial t^2} = 0; \quad \text{For: } \left\{ \begin{array}{l} \mathbf{r}' = \frac{\mathbf{r} - v.\mathbf{t}}{\sqrt{1 - \frac{v^2}{c^2}}} \Rightarrow \frac{\partial r'}{\partial t} = \frac{-v}{\sqrt{1 - \frac{v^2}{c^2}}} \quad \frac{\partial r'}{\partial r} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \\ \mathbf{t}' = \frac{\mathbf{t} - \frac{v}{c^2}.\mathbf{r}}{\sqrt{1 - \frac{v^2}{c^2}}} \Rightarrow \frac{\partial t'}{\partial t} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}; \quad \frac{\partial t'}{\partial r} = \frac{-\frac{v}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}; \end{array} \right.$$

By applying Chain rule for partial derivation, respect to variables r, t :

$$\frac{\partial \varepsilon}{\partial r} = \frac{\partial \varepsilon}{\partial r'} \frac{\partial r'}{\partial r} + \frac{\partial \varepsilon}{\partial t'} \frac{\partial t'}{\partial r} \quad \frac{\partial \varepsilon}{\partial t} = \frac{\partial \varepsilon}{\partial r'} \frac{\partial r'}{\partial t} + \frac{\partial \varepsilon}{\partial t'} \frac{\partial t'}{\partial t}$$

Substituting values previously obtained:

$$\frac{\partial \varepsilon}{\partial r} = \frac{\partial \varepsilon}{\partial r'} \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} + \frac{\partial \varepsilon}{\partial t'} \frac{-\frac{v}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}; \quad \frac{\partial \varepsilon}{\partial t} = \frac{\partial \varepsilon}{\partial r'} \frac{-v}{\sqrt{1 - \frac{v^2}{c^2}}} + \frac{\partial \varepsilon}{\partial t'} \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}};$$

By differentiating again, in order to form all required quadratics components of Wave Equation:

$$\frac{\partial^2 \varepsilon}{\partial r^2} = \frac{1}{(1 - \frac{v^2}{c^2})} \left(\frac{\partial^2 \varepsilon}{\partial r'^2} + \frac{v^2}{c^4} \frac{\partial^2 \varepsilon}{\partial t'^2} - 2 \frac{v}{c^2} \frac{\partial^2 \varepsilon}{\partial r' \partial t'} \right)$$

$$\frac{\partial^2 \varepsilon}{\partial t^2} = \frac{1}{(1 - \frac{v^2}{c^2})} \left(v^2 \frac{\partial^2 \varepsilon}{\partial r'^2} + \frac{\partial^2 \varepsilon}{\partial t'^2} - 2.v. \frac{\partial^2 \varepsilon}{\partial r' \partial t'} \right)$$

And substituting these obtained expressions in Wave Equation, we finally have:

$$\begin{aligned} & \frac{\partial^2 \varepsilon}{\partial r^2} - \frac{1}{c^2} \cdot \frac{\partial^2 \varepsilon}{\partial t^2} = \\ & = \frac{1}{1 - \frac{v^2}{c^2}} \left(\frac{\partial^2 \varepsilon}{\partial r'^2} + \frac{v^2}{c^4} \frac{\partial^2 \varepsilon}{\partial t'^2} - 2 \frac{v}{c^2} \frac{\partial^2 \varepsilon}{\partial r' \partial t'} \right) - \frac{1}{c^2} \cdot \frac{1}{1 - \frac{v^2}{c^2}} \left(v^2 \frac{\partial^2 \varepsilon}{\partial r'^2} + \frac{\partial^2 \varepsilon}{\partial t'^2} - 2.v. \frac{\partial^2 \varepsilon}{\partial r' \partial t'} \right) \\ & \frac{\partial^2 \varepsilon}{\partial r^2} - \frac{1}{c^2} \frac{\partial^2 \varepsilon}{\partial t^2} = \frac{\partial^2 \varepsilon}{\partial r'^2} - \frac{1}{c^2} \frac{\partial^2 \varepsilon}{\partial t'^2} \end{aligned}$$

In this way, it is shown that TVL are consistent with Wave Equation (and with Maxwell Equations). So, Wave Equation under TVL has the same presentation for one and another observer, independent of the path of the moving observer and also independent of the light pulse direction, meeting in such a way Einstein relativistic postulates and being consistent with Maxwell Equations.

The problem that LT have, according to our development, is precisely the following assumptions: $y' = y$ and $z' = z$ that originates $\frac{\partial^2 \varepsilon_x}{\partial y'^2} = \frac{\partial^2 \varepsilon_x}{\partial y^2}$ and $\frac{\partial^2 \varepsilon_x}{\partial z'^2} = \frac{\partial^2 \varepsilon_x}{\partial z^2}$. They don't allow a vectorial treatment through variables r , t . For instance in two dimensions, according to Lorentz, the displacement of light measured by fixed observer at O, is $\mathbf{r} = x \cdot \mathbf{i} + y \cdot \mathbf{j}$, and the corresponding measurement done by moving one at O' is:

$$\mathbf{r}' = x' \cdot \mathbf{i} + y' \cdot \mathbf{j} = \frac{x - v \cdot t}{\sqrt{1 - \frac{v^2}{c^2}}} \mathbf{i} + y \cdot \mathbf{j} = \frac{x}{\sqrt{1 - \frac{v^2}{c^2}}} \mathbf{i} + y \cdot \mathbf{j} - \frac{-v \cdot t}{\sqrt{1 - \frac{v^2}{c^2}}} \mathbf{i}.$$

By observing carefully this last equation, we conclude that it is not possible to obtain an explicit expression of \mathbf{r}' as function of \mathbf{r} and \mathbf{t} . Thus, it can not be possible to obtain an expression for $\frac{\partial r'}{\partial r}$, neither for $\frac{\partial t'}{\partial t}$. In these circumstances we can not continue with the procedure of constructing the vectorial version of the original LT. It can be shown that LT really are not invariant to the Wave equation. Although in some books appears a “demonstration” of the consistency of the LT with Wave Equation, this is not quite general, this is actually a demonstration that is valid only for the particular case of one dimension: the X axis, in where the assumptions cancel out. The chosen example for such demonstration is always presented without any variation: An observer at the origin of the moving system O', which moves on the X axis and a light pulse is sent to the “space” with the usual assumptions. For instance, if this presentation is changed, by establishing that the pulse of light is going parallel to the Z axis, maintaining the moving observer on the X axis, the “demonstration” fails. For showing this, we will work out a known example taken from basic electromagnetic theory:

- (1) Let an electromagnetic plane wave move on Z axis at light speed, $z = ct$, such that the electric field on Y axis, $\varepsilon_y = \varepsilon_o \cdot \sin k \cdot (z - ct)$, depends only on the Z coordinate and time. So, field characteristics will be: $\varepsilon_x = 0$; $\varepsilon_y = \varepsilon_y(z, t)$; $\varepsilon_z = 0$; $x = y = 0$. Suppose that the system O' is moving along the X axis at a velocity v and let's assume, $z' = z$, in order to be under the same premises of LT.

The relationships that hold for this case, according to LT, are:

$$x' = \frac{-v \cdot t}{\sqrt{1 - \frac{v^2}{c^2}}}; \quad x = 0; \quad y' = y = 0; \quad z' = z; \quad t' = \frac{t}{\sqrt{1 - \frac{v^2}{c^2}}} \Rightarrow \frac{\partial x'}{\partial t} = \frac{-v}{\sqrt{1 - \frac{v^2}{c^2}}};$$

$$\frac{\partial t'}{\partial t} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}; \quad \frac{\partial t'}{\partial x} = 0$$

With these premises, we can write: $\frac{\partial \varepsilon_y}{\partial x} = \frac{\partial \varepsilon_y}{\partial y} = 0$, and similarly, $\frac{\partial y'}{\partial t} = 0$; $\frac{\partial t'}{\partial z} = 0$; Given that time is not an explicit variable in the expression of z' , then $\frac{\partial z'}{\partial t} = 0$; and because

$z' = z$, then: $\frac{\partial z'}{\partial z} = 1$. In this way, all the equations corresponding to Wave Equation in function of the coordinate components are reduced to:

$$\frac{\partial^2 \varepsilon_y}{\partial x^2} + \frac{\partial^2 \varepsilon_y}{\partial y^2} + \frac{\partial^2 \varepsilon_y}{\partial z^2} - \frac{1}{c^2} \frac{\partial^2 \varepsilon_y}{\partial t^2} = 0 \quad \Rightarrow \quad \frac{\partial^2 \varepsilon_y}{\partial z^2} - \frac{1}{c^2} \frac{\partial^2 \varepsilon_y}{\partial t^2} = 0$$

Let's try to build the Wave Equation under prime variables. By using the Chain rule we will form components with the prime variables:

$$\begin{aligned} \frac{\partial \varepsilon_y}{\partial z} &= \frac{\partial \varepsilon_y}{\partial x'} \frac{\partial x'}{\partial z} + \frac{\partial \varepsilon_y}{\partial y'} \frac{\partial y'}{\partial z} + \frac{\partial \varepsilon_y}{\partial z'} \frac{\partial z'}{\partial z} + \frac{\partial \varepsilon_y}{\partial t'} \frac{\partial t'}{\partial z} = \frac{\partial \varepsilon_y}{\partial z'} \frac{\partial z'}{\partial z} = \frac{\partial \varepsilon_y}{\partial z'} \quad \Rightarrow \quad \frac{\partial^2 \varepsilon_y}{\partial z^2} = \frac{\partial^2 \varepsilon_y}{\partial z'^2} \\ \frac{\partial \varepsilon_y}{\partial t} &= \frac{\partial \varepsilon_y}{\partial x'} \frac{\partial x'}{\partial t} + \frac{\partial \varepsilon_y}{\partial y'} \frac{\partial y'}{\partial t} + \frac{\partial \varepsilon_y}{\partial z'} \frac{\partial z'}{\partial t} + \frac{\partial \varepsilon_y}{\partial t'} \frac{\partial t'}{\partial t} = \frac{\partial \varepsilon_y}{\partial x'} \frac{\partial x'}{\partial t} + \frac{\partial \varepsilon_y}{\partial t'} \frac{\partial t'}{\partial t} \\ \frac{\partial^2 \varepsilon_y}{\partial t^2} &= \frac{\partial^2 \varepsilon_y}{\partial x'^2} \frac{\partial x'^2}{\partial t^2} + \frac{\partial^2 \varepsilon_y}{\partial t'^2} \frac{\partial t'^2}{\partial t^2} - 2 \cdot \left[\frac{\partial^2 \varepsilon_y}{\partial x' \partial t'} \frac{\partial x'}{\partial t} \frac{\partial t'}{\partial t} \right] \end{aligned}$$

Substituting by their values we obtain:

$\frac{\partial^2 \varepsilon_y}{\partial t^2} = \frac{1}{1 - \frac{v^2}{c^2}} \left(v^2 \frac{\partial^2 \varepsilon_y}{\partial x'^2} + \frac{\partial^2 \varepsilon_y}{\partial t'^2} - 2 \cdot v \cdot \frac{\partial^2 \varepsilon_y}{\partial x' \partial t'} \right)$ Introducing these results, it is obtained a different presentation of the Wave Equation for the prime variables: contrary to what is expected:

$$\frac{\partial^2 \varepsilon_y}{\partial z^2} - \frac{1}{c^2} \frac{\partial^2 \varepsilon_y}{\partial t^2} = \frac{\partial^2 \varepsilon_y}{\partial z'^2} - \frac{1}{c^2} \frac{1}{\left(1 - \frac{v^2}{c^2}\right)} \left(v^2 \frac{\partial^2 \varepsilon_y}{\partial x'^2} + \frac{\partial^2 \varepsilon_y}{\partial t'^2} - 2 \cdot v \cdot \frac{\partial^2 \varepsilon_y}{\partial x' \partial t'} \right)$$

This result shows how the original LT **are not really consistent** with the Maxwell Equations, because it does not preserve the structure of Wave Equation.

(1) Let's do the same job but through the VLT. Expressing the movement of O' and that of the light pulse in a vectorial form, and remembering that components of vector time measured by O are given by the movement of O', we get:

$\alpha = \beta = x = y = t_y = t_z = 0; \Rightarrow \mathbf{t} = t \cdot \mathbf{i}; \mathbf{r} = z \cdot \mathbf{k} \Rightarrow z = r$; By applying:

$$\begin{cases} \mathbf{t}' = k \cdot \left(\mathbf{t} - \frac{v}{c^2} \cdot \mathbf{r} \right) \\ \mathbf{r}' = k \cdot \left(\mathbf{r} - v \cdot \mathbf{t} \right) \end{cases}$$

$$\mathbf{t}' = \frac{t \cdot \mathbf{i} - \frac{v}{c^2} \cdot r \cdot \mathbf{k}}{\sqrt{1 - \frac{v^2}{c^2}}}; \quad \mathbf{r}' = \frac{r \cdot \mathbf{k} - v \cdot t \cdot \mathbf{i}}{\sqrt{1 - \frac{v^2}{c^2}}} \quad \Rightarrow \quad \begin{cases} \frac{\partial t'}{\partial t} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}; & \frac{\partial t'}{\partial r} = \frac{-\frac{v}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}} \\ \frac{\partial r'}{\partial r} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}; & \frac{\partial r'}{\partial t} = \frac{-v}{\sqrt{1 - \frac{v^2}{c^2}}} \end{cases}$$

Wave Equation, in function of r, t had become: $\frac{\partial^2 \varepsilon_y}{\partial r^2} - \frac{1}{c^2} \frac{\partial^2 \varepsilon_y}{\partial t^2} = 0$. Operating as before, and substituting values, primed Wave Equation is consistently obtained:

$$\begin{aligned} \frac{\partial^2 \varepsilon}{\partial r^2} - \frac{1}{c^2} \frac{\partial^2 \varepsilon}{\partial t^2} &= \left[\frac{\partial^2 \varepsilon}{\partial r'^2} \frac{\partial r'^2}{\partial r^2} + \frac{\partial^2 \varepsilon}{\partial t'^2} \frac{\partial t'^2}{\partial t^2} - 2 \cdot \frac{\partial^2 \varepsilon}{\partial r' \partial t'} \frac{\partial r'}{\partial r} \frac{\partial t'}{\partial t} \right] \\ - \frac{1}{c^2} \left[\frac{\partial^2 \varepsilon}{\partial r'^2} \frac{\partial r'^2}{\partial t^2} + \frac{\partial^2 \varepsilon}{\partial t'^2} \frac{\partial t'^2}{\partial t^2} - 2 \cdot \frac{\partial^2 \varepsilon}{\partial r' \partial t'} \frac{\partial r'}{\partial r} \frac{\partial t'}{\partial t} \right] & \frac{\partial^2 \varepsilon}{\partial r^2} - \frac{1}{c^2} \frac{\partial^2 \varepsilon}{\partial t^2} = \frac{\partial^2 \varepsilon}{\partial r'^2} - \frac{1}{c^2} \frac{\partial^2 \varepsilon}{\partial t'^2} \end{aligned}$$

As so it was expected. This also means that VLT are consistent with Wave Equation and in general with Maxwell Equations.

Annex 3

Vectorial Lorentz Transformations Applied To Curvilinear Movement

As we will demonstrate next, it is possible to apply the new given approach to the Lorentz Transformations, the Vectorial Lorentz Transformations (VLT), to an inertial system of coordinates with curvilinear movement, with respect to a fixed system, located in a point throughout the curvilinear trajectory of the moving system.

We can establish that, inertial systems are not only those with null acceleration, but those where “the sum of acting forces is null”. These include not only those with null acceleration in rectilinear movement, but also those in curvilinear movement with a constant Angular Momentum.

For the movement of Earth around the Sun, the summation of the gravitational force of Sun onto Earth plus the Earth’s centrifugal force gives a null result, reason why the Earth movement is inertial according to since we have defined it. In this way, earth’s movement is neither impeded nor eased by any additional external force. We will try to reproduce this movement in Fig. A6, where the first observer is on the moving system, Earth, at O' , and the second observer will be fixed on the elliptic path at the nearest point to the Sun, the perihelion.

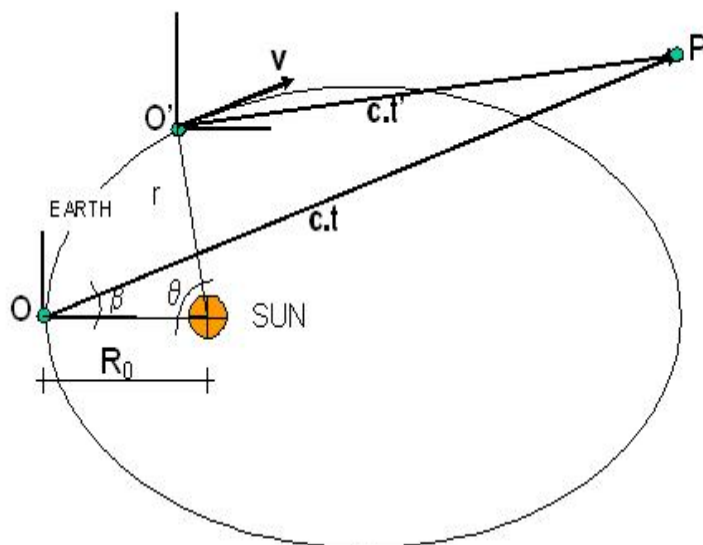


Fig. A6

Let’s denote R_0 , as the distance between Sun And Earth at the moment when observers start measuring the movement, and r , the generic position of Earth.

By taking a closer view of this movement, for two dimensions, see Fig. A7, let’s establish that, when O' and O coincide, a pulse of light is sent forming an angle β with X axis, see Fig. A6, and an angle γ between the tangential velocity v of O' with Y axis as it is shown in Fig. A7. Let’s equally define θ , as the angle swept by radius r from $r = R_0$,

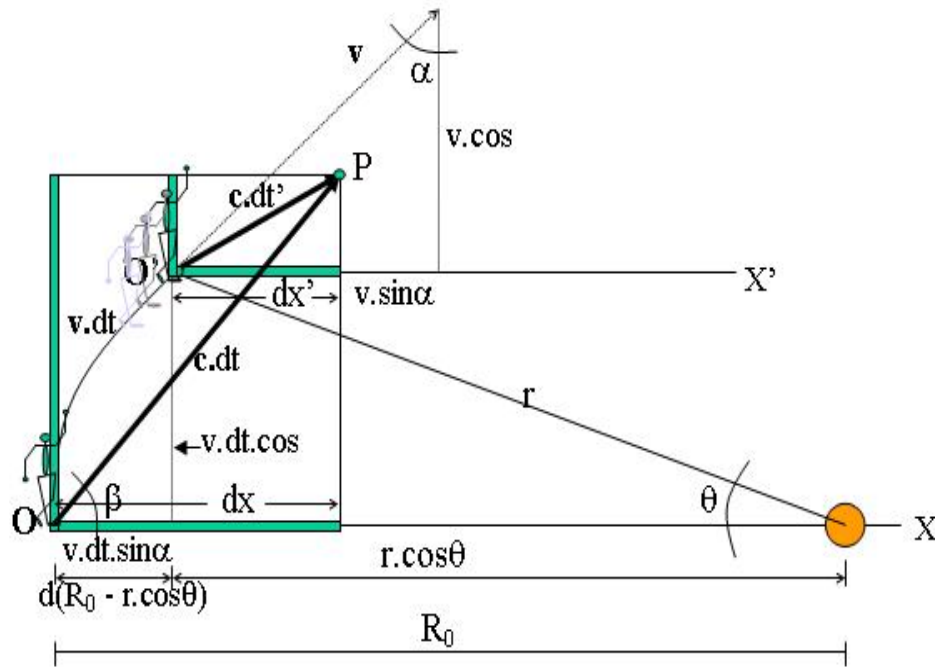


Fig. N° A7

to the new position r of the moving observer after a period of time dt . At this moment light pulse has reached point P . From figures A6 and A7, let's establish the following relationships:

$$dx' = k(dx - v.dt. \sin \gamma) \quad dy' = k(dy - v.dt. \cos \gamma) \quad (A3.1)$$

From the same figure we can establish:

$$v.dt. \sin \gamma = d(R_0 - r. \cos \theta) \quad v.dt. \cos \gamma = d(R_0 - r. \sin \theta) \quad (A3.2)$$

Given that the light speed is the same measured by any observer, it must be met:

$$dx^2 + dy^2 = c^2.dt^2 \quad dx'^2 + dy'^2 = c^2.dt'^2 \quad (A3.3)$$

Substituting dx' , dy' , by their expressions (A3.1) and (A3.2) into (A3.3), similar expressions previously obtained for rectilinear movement are achieved:

$$dx' = \frac{dx - v.dt. \sin \gamma}{\sqrt{1 - \frac{v^2}{c^2}}} \quad dy' = \frac{dy - v.dt. \cos \gamma}{\sqrt{1 - \frac{v^2}{c^2}}} \quad dt' = dt. \frac{\sqrt{(\sin \gamma - \frac{v.u_x}{c^2})^2 + (\cos \gamma - \frac{v.u_y}{c^2})^2}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$u'_x = \frac{u_x - v. \sin \gamma}{\sqrt{(\sin \gamma - \frac{v.u_x}{c^2})^2 + (\cos \gamma - \frac{v.u_y}{c^2})^2}} \quad u'_y = \frac{u_y - v. \cos \gamma}{\sqrt{(\sin \gamma - \frac{v.u_x}{c^2})^2 + (\cos \gamma - \frac{v.u_y}{c^2})^2}} \quad (A3.4)$$

Namely,

$$d\mathbf{r}' = \frac{d\mathbf{r} - v \cdot d\mathbf{t}}{\sqrt{1 - \frac{v^2}{c^2}}}; \quad dt' = \frac{dt - \frac{v}{c^2} d\mathbf{r}}{\sqrt{1 - \frac{v^2}{c^2}}} \quad \mathbf{u}' = \frac{d\mathbf{r}'}{dt'} \quad (\text{A3.5})$$

These results show that the structure of differential VLT for curvilinear is the same previously viewed for rectilinear movement. Obviously, all this indicates that application of LLT to curvilinear movement is also valid.