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Magnetized Bianchi Type VI_0 Barotropic Massive String Universe with Decaying Vacuum Energy Density Λ

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Abstract: Bianchi type VI_0 massive string cosmological models using the technique given by Letelier (1983) with magnetic field are investigated. To get the deterministic models, we assume that the expansion (θ) in the model is proportional to the shear (σ) and also the fluid obeys the barotropic equation of state. It was found that vacuum energy density $\Lambda \propto \frac{1}{t^2}$ which matches with natural units. The behaviour of the models from physical and geometrical aspects in presence and absence of magnetic field is also discussed.

Keywords: Massive String, Bianchi type VI_0 Universe, Magnetic Field
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1. Introduction

The problem of the cosmological constant is one of the most salient and unsettled problems in cosmology. The smallness of the effective cosmological constant recently observed ($\Lambda_0 \leq 10^{-56}\text{cm}^{-2}$) constitutes the most difficult problems involving cosmology and elementary particle physics theory. To explain the striking cancelation between the "bare" cosmological constant and the ordinary vacuum energy contributions of the quantum fields, many mechanisms have been proposed during last few years [1]. The "cosmological constant problem" can be expressed as the discrepancy between the negligible value of Λ has for the present universe (as can be seen by the successes of Newton's theory of gravitation [2]) and the values 10^{50} larger expected by the Glashow-Salam-Weinberg

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model [3] or by grand unified theory (GUT) where it should be 10^{107} larger [4]. The cosmological term Λ is then small at the present epoch. The problem in this approach is to determine the right dependence of Λ upon S or t. Recent observations of Type Ia supernovae (Perlmutter et al. [5], Riess et al. [6]) and measurements of the cosmic microwave background [7] suggest that the universe is an accelerating expansion phase [8].

Several ansätz have been proposed in which the Λ term decays with time (see Refs. Gasperini [9], Berman [10]−[12], Berman et al. [13]−[15], Freese et al. [16], Özer and Taha [17], Ratra and Peebles [18], Chen and Hu [19], Abdussattar and Vishwakarma [20], Gariel and Le Denmat [21], Pradhan et al. [22]). Of the special interest is the ansätz $\Lambda \propto S^{-2}$ (where S is the scale factor of the Robertson-Walker metric) by Chen and Wu [19], which has been considered/modified by several authors (Abdel-Rahaman [23], Carvalho et al. [24], Silveira and Waga [25], Vishwakarma [26]).

One of the outstanding problems in cosmology today is developing a more precise understanding of structure formation in the universe, that is, the origin of galaxies and other large-scale structures. Existing theories for the structure formation of the Universe fall into two categories, based either upon the amplification of quantum fluctuations in a scalar field during *inflation*, or upon symmetry breaking phase transition in the early Universe which leads to the formation of *topological defects* such as domain walls, cosmic strings, monopoles, textures and other 'hybrid' creatures. Cosmic strings play an important role in the study of the early universe. These arise during the phase transition after the big bang explosion as the temperature goes down below some critical temperature as predicted by grand unified theories (see Refs. Zel'dovich et al. [27], Kibble [28, 29], Everett [30], Vilenkin [31]). It is believed that cosmic strings give rise to density perturbations which lead to formation of galaxies (Zel'dovich [32]). These cosmic strings have stress energy and couple to the gravitational field. Therefore, it is interesting to study the gravitational effect which arises from strings. The general treatment of strings was initiated by Letelier [33, 34] and Stachel [35].

The occurrence of magnetic fields on galactic scale is well-established fact today, and their importance for a variety of astrophysical phenomena is generally acknowledged. Several authors (Zeldovich [36], Harrison [37], Misner, Thorne and Wheeler [38], Asseo and Sol [39], Pudritz and Silk [40], Kim, Tribble, and Kronberg [41], Perley, and Taylor [42], Kronberg, Perry, and Zukowski [43], Wolfe, Lanzetta and Oren [44], Kulsrud, Cen, Ostriker and Ryu [45] and Barrow [46]) have pointed out the importance of magnetic field in different context. As a natural consequences, we should include magnetic fields in the energy-momentum tensor of the early universe. The string cosmological models with a magnetic field are also discussed by Benerjee et al. [47], Chakraborty [48], Tikekar and Patel ([49, 50], Patel and Maharaj [51] Singh and Singh [52].

Recently, Bali et al. [53]−[57], Pradhan et al. [58] − [60], Yadav et al. [61] and Pradhan [62] have investigated Bianchi type I, II, III, V,IX and cylindrically symmetric magnetized string cosmological models in presence and absence of magnetic field. Tikekar and Patel [50] have investigated some solutions for Bianchi type VI_0 cosmology in presence and absence of magnetic field. In this paper we have derived some Bianchi type VI_0 string cosmological models for perfect fluid distribution in presence and absence of magnetic field and discussed the variation of Λ with time. This paper is organized as follows: The metric and field equations are presented in Section 2. In Section 3, we deal with the solution of the field equations in presence of magnetic field. In Section 4, we have described some geometric and physical behavior of the model. Section 5 includes the solution in absence of magnetic field. In Section 6, we have discussed the variation of cosmological constant Λ with time in presence and absence of magnetic field. In the last Section 7, concluding remarks are given.

2. The Metric and Field Equations

We consider the Bianchi Type VI_0 metric in the form

$$ds^2 = -dt^2 + A^2(t)dx^2 + B^2(t)e^{2x}dy^2 + C^2(t)e^{-2x}dz^2. \quad (1)$$

The energy-momentum tensor for a cloud of strings in presence of magnetic field is taken into the form

$$T_{ik} = (\rho + p)v_i v_k + p g_{ik} - \lambda x_i x_k + [g^{lm}F_{il}F_{km} - \frac{1}{4}g_{ik}F_{lm}F^{lm}], \quad (2)$$

where v_i and x_i satisfy conditions

$$v^i v_i = -x^i x_i = -1, \;\; v^i x_i = 0. \quad (3)$$

In equations (2), p is isotropic pressure, ρ is rest energy density for a cloud strings, λ is the string tension density, F_{ij} is the electromagnetic field tensor, x^i is a unit space-like vector representing the direction of string, and v^i is the four velocity vector satisfying the relation

$$g_{ij}v^i v^j = -1. \quad (4)$$

Here, the co-moving coordinates are taken to be $v^1 = 0 = v^2 = v^3$ and $v^4 = 1$ and $x^i = (\frac{1}{A}, 0, 0, 0)$. The Maxwell's equations

$$F_{ij;k} + F_{jk;i} + F_{ki;j} = 0, \quad (5)$$

$$F^{ik}_{;k} = 0, \quad (6)$$

are satisfied by

$$F_{23} = K(\text{say}) = \text{constant}, \quad (7)$$

where a semicolon (;) stands for covariant differentiation.

The Einstein's field equations (with $\frac{8\pi G}{c^4}=1$)

$$R_i^j - \frac{1}{2}Rg_i^j = -T_i^j - \Lambda g_i^j, \tag{8}$$

for the line-element(1) lead to the following system of equations:

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{B_4C_4}{BC} + \frac{1}{A^2} = -\left[p - \lambda - \frac{K^2}{2B^2C^2}\right] - \Lambda, \tag{9}$$

$$\frac{A_{44}}{A} + \frac{C_{44}}{C} + \frac{A_4C_4}{AC} - \frac{1}{A^2} = -\left[p + \frac{K^2}{2B^2C^2}\right] - \Lambda, \tag{10}$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4B_4}{AB} - \frac{1}{A^2} = -\left[p + \frac{K^2}{2B^2C^2}\right] - \Lambda, \tag{11}$$

$$\frac{A_4B_4}{AB} + \frac{B_4C_4}{BC} + \frac{C_4A_4}{CA} - \frac{1}{A^2} = \left[\rho + \frac{K^2}{2B^2C^2}\right] - \Lambda, \tag{12}$$

$$\frac{1}{A}\left[\frac{C_4}{C} - \frac{B_4}{B}\right] = 0, \tag{13}$$

where the sub indice 4 in A, B, C denotes ordinary differentiation with respect to t. The velocity field v^i is irrotational. The scalar expansion θ and components of shear σ_{ij} are given by

$$\theta = \frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}, \tag{14}$$

$$\sigma_{11} = \frac{A^2}{3}\left[\frac{2A_4}{A} - \frac{B_4}{B} - \frac{C_4}{C}\right], \tag{15}$$

$$\sigma_{22} = \frac{B^2}{3}\left[\frac{2B_4}{B} - \frac{A_4}{A} - \frac{C_4}{C}\right], \tag{16}$$

$$\sigma_{33} = \frac{C^2}{3}\left[\frac{2C_4}{C} - \frac{A_4}{A} - \frac{B_4}{B}\right], \tag{17}$$

$$\sigma_{44} = 0. \tag{18}$$

Therefore

$$\sigma^2 = \frac{1}{2}\left[(\sigma^1{}_1)^2 + (\sigma^2{}_2)^2 + (\sigma^3{}_3)^2 + (\sigma^4{}_4)^2\right],$$

which leads to

$$\sigma^2 = \frac{1}{3}\left[\frac{A_4^2}{A^2} + \frac{B_4^2}{B^2} + \frac{C_4^2}{C^2} - \frac{A_4B_4}{AB} - \frac{B_4C_4}{BC} - \frac{C_4A_4}{CA}\right].$$

Above relation after using (13) reduces to

$$\sigma = \frac{1}{\sqrt{3}}\left(\frac{A_4}{A} - \frac{B_4}{B}\right). \tag{19}$$

3. Solutions of the Field Equations

The field equations (9)-(13) are a system of five equations with seven unknown parameters A, B, C, ρ, p, λ and Λ. We need two additional conditions to obtain explicit solutions of the system.

Equation (13) leads to

$$C = mB, \tag{20}$$

where m is an integrating constant.

We first assume that the expansion (θ) in the model is proportional to shear (σ). The motive behind assuming this condition is explained with reference to Thorne [63], the observations of the velocity-red-shift relation for extragalactic sources suggest that Hubble expansion of the universe is isotropic today within ≈ 30 per cent [64, 65]. To put more precisely, red-shift studies place the limit

$$\frac{\sigma}{H} \leq 0.3$$

on the ratio of shear, σ, to Hubble constant, H, in the neighborhood of our Galaxy today. Collins et al. [66] have pointed out that for spatially homogeneous metric, the normal congruence to the homogeneous expansion satisfies that the condition $\frac{\sigma}{\theta}$ is constant. This condition and Eq. (20) lead to

$$\frac{1}{\sqrt{3}}\left(\frac{A_4}{A} - \frac{B_4}{B}\right) = l\left(\frac{A_4}{A} + \frac{2B_4}{B}\right) \tag{21}$$

which yields to

$$\frac{A_4}{A} = n\frac{B_4}{B}, \tag{22}$$

where $n = \frac{(2l\sqrt{3}+1)}{(1-l\sqrt{3})}$ and l are constants. Eq. (22), after integration, reduces to

$$A = \beta B^n, \tag{23}$$

where β is a constant of integration. Eqs. (10) and (12) lead to

$$p = -\frac{K^2}{2B^2C^2} - \left(\frac{A_{44}}{A} + \frac{C_{44}}{C} + \frac{A_4C_4}{AC} - \frac{1}{A^2}\right) - \Lambda, \tag{24}$$

and

$$\rho = \frac{A_4B_4}{AB} + \frac{B_4C_4}{BC} + \frac{C_4A_4}{CA} - \frac{1}{A^2} - \frac{K^2}{2B^2C^2} + \Lambda, \tag{25}$$

respectively. Now let us consider that the fluid obeys the barotropic equation of state

$$p = \gamma\rho, \tag{26}$$

where $\gamma(\gamma \leq 0 \leq 1)$ is a constant. Eqs. (24) to (26) lead to

$$\frac{A_{44}}{A} + \frac{C_{44}}{C} + (1+\gamma)\frac{A_4C_4}{AC} + \gamma\left(\frac{A_4B_4}{AB} + \frac{B_4C_4}{BC}\right) - (1+\gamma)\frac{1}{A^2} +$$
$$(1-\gamma)\frac{K^2}{2B^2C^2} + (1+\gamma)\Lambda = 0. \tag{27}$$

Eq. (27) with the help of (20) and (23) reduces to

$$2B_{44} + \frac{2(n^2+2\gamma n+\gamma)}{(n+1)}\frac{B_4^2}{B^2} = \frac{2(1+\gamma)}{\beta^2 B^{2n-1}} + \frac{(1-\gamma)K^2}{m^2B^3} + 2l_0 B, \tag{28}$$

where $l_0 = (1+\gamma)\Lambda$.
Let us consider $B_4 = f(B)$ and $f' = \frac{df}{dB}$. Hence Eq. (28) takes the form

$$\frac{d}{df}(f^2) + \frac{2\alpha}{B}f^2 = \frac{2(1+\gamma)}{\beta^2 B^{2n-1}} + \frac{(1-\gamma)K^2}{m^2B^3} + 2l_0 B, \tag{29}$$

where $\alpha = \frac{(n^2+2n\gamma+\gamma)}{(n+1)}$. Eq. (29) after integrating reduces to

$$f^2 = \frac{2(1+\gamma)B^{-2n+2}}{\beta^2(2\alpha-2n+2)} + \frac{(1-\gamma)K^2}{2m^2(\alpha-1)} + \frac{l_0B^2}{(\alpha+1)} + MB^{-2\alpha}, \quad \gamma \neq 1, \tag{30}$$

where M is an integrating constant. To get deterministic solution in terms of cosmic string t, we suppose $M = 0$ without any loss of generality. In this case Eq. (30) takes the form

$$f^2 = aB^{-2(n-1)} + bB^{-2} + kB^2, \tag{31}$$

where

$$a = \frac{2(1+\gamma)}{\beta^2(2\alpha-2n+2)}, \quad b = \frac{(1-\gamma)K^2}{2m^2(\alpha-1)}, \quad k = \frac{(1+\gamma)\Lambda}{(\alpha+1)}.$$

Therefore, we have

$$\frac{dB}{\sqrt{aB^{-2(n-1)} + bB^{-2} + kB^2}} = dt. \tag{32}$$

To get deterministic solution, we assume $n = 2$. In this case integrating Eq. (32), we obtain

$$B^2 = \sqrt{(a+b)}\frac{\sinh(2\sqrt{k}t)}{\sqrt{k}}. \tag{33}$$

Hence, we have

$$C^2 = m^2\sqrt{(a+b)}\frac{\sinh(2\sqrt{k}t)}{\sqrt{k}}, \tag{34}$$

$$A^2 = \beta^2(a+b)\frac{\sinh^2(2\sqrt{k}t)}{k}, \tag{35}$$

where $k > 0$ without any loss of generality.

Therefore, the metric (1), in presence of magnetic field, reduces to the form

$$ds^2 = -dt^2 + \beta^2(a+b)\ \frac{\sinh^2(2\sqrt{k}t)}{k}\ dx^2 +$$
$$\sqrt{(a+b)}\ \frac{\sinh(2\sqrt{k}t)}{\sqrt{k}}\ e^{2x}\ dy^2 + m^2\sqrt{(a+b)}\ \frac{\sinh(2\sqrt{k}t)}{\sqrt{k}}\ e^{-2x}\ dz^2. \tag{36}$$

4. The Geometric and Physical Significance of Model

The pressure (p), energy density (ρ), the string tension density (λ), the particle density (ρ_p), the scalar of expansion (θ), the shear tensor (σ) and the proper volume (V^3) for the model (36) are given by

$$p = \left[\frac{k}{\beta^2(a+b)} - \frac{K^2 k}{2m^2(a+b)}\right]\coth^2(2\sqrt{k}t) +$$
$$\left[\frac{K^2}{2m(a+b)} - \frac{1}{\beta^2(a+b)} - 8\right]k - \Lambda, \tag{37}$$

$$\rho = \left[5k - \frac{k}{(a+b)}\left(\frac{K^2}{2m^2} + \frac{1}{\beta^2}\right)\right]\coth^2(2\sqrt{k}t) +$$
$$\frac{k}{(a+b)}\left(\frac{K^2}{2m^2} + \frac{1}{\beta^2}\right) + \Lambda, \tag{38}$$

where $p = \gamma\rho$ is satisfied by (27).

$$\lambda = \left[\frac{2k}{\beta^2(a+b)} - \frac{K^2 k}{m^2(a+b)} - k\right]\coth^2(2\sqrt{k}t) +$$
$$\left\{\frac{K^2 k}{m^2(a+b)} - \frac{2k}{\beta^2(a+b)} - 4k\right\}, \tag{39}$$

$$\rho_p = \rho - \lambda = \left[\frac{K^2 k}{2m^2(a+b)} - \frac{3k}{\beta^2(a+b)} + k\right]\coth^2(2\sqrt{k}t)$$
$$+9k + \left\{\frac{3k}{\beta^2(a+b)} - \frac{K^2}{2m^2(a+b)}\right\}, \tag{40}$$

$$\theta = 4\sqrt{k}\coth(2\sqrt{k}t), \tag{41}$$

$$\sigma = \sqrt{\frac{k}{3}}\coth(2\sqrt{k}t), \tag{42}$$

$$V^3 = \frac{\beta m(a+b)}{k}\sinh^2(2\sqrt{k}t). \tag{43}$$

From Eqs. (30) and (31), we obtain

$$\frac{\sigma}{\theta} = \text{constant}. \tag{44}$$

The deceleration parameter is given by

$$q = -\frac{\ddot{R}/R}{\dot{R}^2/R^2} = -\left[\frac{\frac{8k}{3} - \frac{8k}{9}\coth^2(2\sqrt{k}t)}{\frac{16k}{9}\coth^2(2\sqrt{k}t)}\right]. \tag{45}$$

From (45), we observe that

$$q < 0 \quad if \quad \coth^2(2\sqrt{k}t) < 3$$

and

$$q > 0 \quad if \quad \coth^2(2\sqrt{k}t) > 3.$$

From (38), $\rho \geq 0$ implies that

$$\coth^2(2\sqrt{k}t) \leq \left[\frac{\frac{k}{(a+b)}\left(\frac{K^2}{2m^2} + \frac{1}{\beta^2}\right) + \Lambda}{\frac{k}{(a+b)}\left(\frac{K^2}{2m^2} + \frac{1}{\beta^2}\right) - 5k}\right]. \tag{46}$$

Also from (40), $\rho_p \geq 0$ implies that

$$\coth^2(2\sqrt{k}t) \leq \left[\frac{\frac{3k}{\beta^2(a+b)} - \frac{K^2}{2m^2(a+b)} + 9k}{\frac{3k}{\beta^2(a+b)} - \frac{K^2 k}{2m^2(a+b)} - k}\right]. \tag{47}$$

Thus the energy conditions $\rho \geq 0$, $\rho_p \geq 0$ are satisfied under conditions given by (46) and (47).

The model (36) starts with a big bang at $t = 0$. The expansion in the model decreases as time increases. The proper volume of the model increases as time increases. Since $\frac{\sigma}{\theta}$ = constant, hence the model does not approach isotropy. Since ρ, λ, θ, σ tend to infinity and $V^3 \to 0$ at initial epoch $t = 0$, therefore, the model (36) for massive string in presence of magnetic field has Line-singularity (Banerjee et al. [47]). For the condition $\coth^2(2\sqrt{k}t) < 3$, the solution gives accelerating model of the universe. It can be easily seen that when $\coth^2(2\sqrt{k}t) > 3$, our solution represents decelerating model of the universe.

5. Solutions in Absence of Magnetic Field

In absence of magnetic field, i.e. when $b \to 0$ i.e. $K \to 0$, we obtain

$$B^2 = 2\sqrt{2}\,\frac{\sinh(2\sqrt{k}t)}{2\sqrt{k}},$$

$$C^2 = 2m^2\sqrt{a}\,\frac{\sinh(2\sqrt{k}t)}{2\sqrt{k}},$$

$$A^2 = 4a\beta^2\,\frac{\sinh^2(2\sqrt{k}t)}{4k}. \tag{48}$$

Hence, in this case, the geometry of the universe (36) reduces to

$$ds^2 = -dt^2 + 4\beta^2 a\,\frac{\sinh^2(2\sqrt{k}t)}{4k}\,dx^2 +$$

$$2\sqrt{2}\,\frac{\sinh(2\sqrt{k}t)}{2\sqrt{k}}\,e^{2x}\,dy^2 + 2m^2\sqrt{a}\,\frac{\sinh(2\sqrt{k}t)}{2\sqrt{k}}\,e^{-2x}\,dz^2. \tag{49}$$

The pressure (p), energy density (ρ), the string tension density (λ), the particle density (ρ_p), the scalar of expansion (θ), the shear tensor (σ) and the proper volume (V^3) for the model (49) are given by

$$p = \frac{k}{a\beta^2} \coth^2 (2\sqrt{k}t) - \left(\frac{1}{a\beta^2} + 8 \right) k - \Lambda, \tag{50}$$

$$\rho = \left(5k - \frac{k}{a\beta^2} \right) \coth^2 (2\sqrt{k}t) + \frac{k}{a\beta^2} + \Lambda, \tag{51}$$

$$\lambda = \left[\frac{2k}{a\beta^2} - k \right] \coth^2 (2\sqrt{k}t) - \left\{ \frac{2k}{a\beta^2} + 4k \right\}, \tag{52}$$

$$\rho_p = \rho - \lambda = \left[k - \frac{3k}{a\beta^2} \right] \coth^2 (2\sqrt{k}t) + 9k + \frac{3k}{\beta^2 a}, \tag{53}$$

$$\theta = 4\sqrt{k} \coth (2\sqrt{k}t), \tag{54}$$

$$\sigma = \sqrt{\frac{k}{3}} \coth (2\sqrt{k}t), \tag{55}$$

$$V^3 = \frac{\beta m a}{k} \sinh^2 (2\sqrt{k}t). \tag{56}$$

From Eqs. (54) and (55), we obtain

$$\frac{\sigma}{\theta} = \text{constant}. \tag{57}$$

From (51), $\rho \geq 0$ implies that

$$\coth^2 (2\sqrt{k}t) \leq \left[\frac{\frac{k}{a\beta^2} + \Lambda}{\frac{k}{a\beta^2} - 5k} \right]. \tag{58}$$

Also from (53), $\rho_p \geq 0$ implies that

$$\coth^2 (2\sqrt{k}t) \leq \left[\frac{\frac{3k}{a\beta^2} + ak}{\frac{3k}{a\beta^2} - k} \right]. \tag{59}$$

Thus the energy conditions $\rho \geq 0$, $\rho_p \geq 0$ are satisfied under conditions given by (58) and (59).

The model (49) starts with a big bang at $t = 0$ and the expansion in the model decreases as time increases. The spatial volume of the model increases as time increases. Since $\frac{\sigma}{\theta}$ = constant, hence the anisotropy is maintained throughout. Since ρ, λ, θ, σ tend to infinity and $V^3 \to 0$ at initial epoch $t = 0$, therefore, the model (49) for massive string in absence of magnetic field has Line-singularity [47].

6. Variation of Λ with time

Equations (37) and (38) with the use of (26) reduce to

$$\coth^2(2\sqrt{k}t) = \left[\frac{\ell+\alpha+1}{\ell-5\gamma}\right], \tag{60}$$

where

$$\ell = \frac{(1+\gamma)}{\beta^2(a+b)} - \frac{K^2(1-\gamma)}{2m^2(a+b)}. \tag{61}$$

Thus ℓ decreases as magnetic field increases. From equation (60), we obtain

$$2\sqrt{k}t = \coth^{-1}\left[\frac{\ell+\alpha+1}{\ell-5\gamma}\right]^{\frac{1}{2}}, \tag{62}$$

where $\ell+\alpha+1 > \ell-5\gamma$ implies that $\alpha+1+5\gamma > 0$ which is true. Putting the value of k in (62), we obtain

$$\sqrt{\Lambda}t = \frac{\sqrt{1+\alpha}}{2\sqrt{1+\gamma}}\coth^{-1}\left[\frac{\ell+\alpha+1}{\ell-5\gamma}\right]^{\frac{1}{2}} = \text{constant}, \tag{63}$$

which implies that

$$\Lambda = \frac{L}{t^2}, \tag{64}$$

where L is constant. Here we observe that when $t \to 0$ then $\Lambda \to \infty$ and when $t \to \infty$ then $\Lambda \to 0$. Here $\Lambda \propto \frac{1}{t^2}$ which gives fundamental condition supported by observations.

In absence of magnetic field i.e. when $K \to 0$ then

$$\ell \to \frac{(1+\gamma)}{\beta^2(a+b)} = \text{s (say)}. \tag{65}$$

In this case equations (37) and (38) with the use of (26) reduce to

$$\coth^2(2\sqrt{k}t) = \left[\frac{s+\alpha+1}{s-5\gamma}\right], \tag{66}$$

Putting the value of k in (66), we obtain

$$\sqrt{\Lambda}t = \frac{\sqrt{1+\alpha}}{2\sqrt{1+\gamma}}\coth^{-1}\left[\frac{s+\alpha+1}{s-5\gamma}\right]^{\frac{1}{2}} = \text{constant}, \tag{67}$$

which implies that

$$\Lambda = \frac{Q}{t^2}, \tag{68}$$

where Q is constant. Here we observe that when $t \to 0$ then $\Lambda \to \infty$ and when $t \to \infty$ then $\Lambda \to 0$. Here $\Lambda \propto \frac{1}{t^2}$ which gives fundamental condition supported by observations. A number of authors have argued in favor of the dependence $\Lambda \to t^{-2}$ in different context. It has also be found, by several authors, that when one supposes variable gravitational and cosmological "constant" in Brans-Dicke theories one finds the relation like equations

(64) and (68). Berman and Som [13] pointed out that the relation $\Lambda \rightarrow t^{-2}$ seems to play a major role in cosmology. In fact, Berman, Som, and Gomide [14] found this relation in Brans-Dicke static models; Berman [10] found it in a static universe with Endo-Fukui modified Brans-Dicke cosmology; Berman and Som [13] found it again in general Brans-Dicke models which obey the perfect gas equation of state [11, 12]. Berman [15] also found this relation in general relativity. We have derived the same variation of Λ with time in massive string cosmology in this article.

7. Concluding Remarks

Some Bianchi type VI_0 massive string cosmological models with a perfect fluid as the source of matter are obtained in presence and absence of magnetic field. Generally, the models are expanding, shearing and non-rotating. In presence of perfect fluid it represents an accelerating universe during the span of time mentioned below equation (45) as decelerating factor $q < 0$ and it represents decelerating universe as $q > 0$. All the two massive string cosmological models obtained in the present study have Line-singularity (Banerjee et al. [47]) at the initial epoch $t = 0$. The variation of cosmological term in presence and absence of magnetic field is consistent with recent observations. To solve the age parameter and density parameter, one requires the cosmological constant to be positive or equivalently the deceleration parameter to be negative. The nature of the cosmological constant Λ and the energy density ρ have been examined. We have found that the cosmological parameter Λ varies inversely with the square of time, which matches its natural units. This supports the views in favour of the dependence $\Lambda \rightarrow t^{-2}$ first expressed by Bertolami [67, 68] and later on observed by several authors [9]−[22]. The density is easily adjustable to what we observe today, so that there is no need to have recourse to any critical density, and the $\Lambda \rightarrow t^{-2}$ law guarantees that we may explain why the present value of Λ is negligible in comparison with the early universe values as required by particle physics.

We have also observed that the magnetic field gives positive contribution to expansion, shear and the free gravitational field which die out for large value of t at a slower rate than the corresponding quantities in the absence of magnetic field.

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